Chapter 11

Preferred and Non-Preferred Reference Frames

11.1 Preferred Reference Frame

In this sub-chapter the preferred reference frame Σ' is shortly stated. In this frame Σ' the metric is the pseudo-Euclidean geometry, i.e.

$$(cd\tau')^2 = (ds')^2 = -\eta_{kl}{}'dx^{k\prime}dx^{l\prime} \tag{11.1a}$$

with

$$\eta_{11}{}' = \eta_{22}{}' = \eta_{33}{}' = 1, \eta'_{44} = -1, \ \eta'_{ij} = 0 \ (\text{i}\neq\text{j}). \tag{11.1b}$$

In addition, the inverse tensor $\eta^{ij\prime}$ is given by

$$\eta'_{ik}\eta^{kj} = \delta_i^j. \tag{11.2a}$$

It follows

$$\eta^{11\prime} = \eta^{22\prime} = \eta^{33\prime} = 1, \eta^{44\prime} = -1, \eta^{ij\prime} = 0 \ (\text{i}\neq\text{j}). \tag{11.2b}$$

Let $w' = \left(w^1, w^{2\prime}, w^{2\prime}\right)$ be a constant velocity vector and put

$$\gamma = \left(1 - \left|\frac{w\prime}{c}\right|^2\right)^{-1/2}. \tag{11.3}$$

Then, the Lorentz-transformations

$$\begin{aligned} \tilde{x}^{i\prime} &= x^{i\prime} + (\gamma - 1)\frac{(x', w\prime)}{|w\prime|^2}w^{i\prime} + \gamma t' w^{i\prime} \ (\text{i}=1, 2, 3) \\ c\tilde{t}' &= \gamma\left(ct' + \left(x', \frac{w\prime}{c}\right)\right) \end{aligned} \tag{11.4}$$

do not change the line-element (11.1). All the quantities in Σ' are subsequently denoted with a prime and we put

$$x^{4\prime} = ct'. \tag{11.5}$$

The inverse formulae are

$$\begin{aligned} x^{i\prime} &= \tilde{x}^{i\prime} + (\gamma - 1)\frac{(\tilde{x}', w')}{|w'|^2}w^{i\prime} - \gamma\tilde{t}'w^{i\prime} \quad (i = 1, 2, 3) \\ x^{4\prime} &= \gamma\left(\tilde{x}^{4\prime} - \left(\tilde{x}', \frac{w'}{c}\right)\right). \end{aligned} \tag{11.6}$$

Hence, the transformations (11.4) and (11.6) give the possibility to transform a known event in Σ' into the same event moving with constant velocity w' in Σ'.

These are the well-known results of special relativity but the transformations (11.4) and (11.6) are always in in the same frame Σ' in contrast to the interpretation of special relativity where the transformations give the same result in a uniformly moving frame with velocity w'.The light velocity in the preferred frame Σ' is always the vacuum light velocity c.

11.2 Non-Preferred Reference Frame

Let us now consider a reference frame Σ which moves with velocity $-v' = -(v^{1\prime}, v^{2\prime}, v^{3\prime})$ relative to the preferred frame Σ'. All the results of this sub-chapter can be found in the article [Pet 86].

The non-preferred reference frame Σ is received from the preferred frame Σ' by the transformations

$$x^i = x^{i\prime} \text{ (i=1,2,3)}, \; x^4 = x^{4\prime} - \left(x', \frac{v\prime}{c}\right). \tag{11.7a}$$

The inverse transformation is

$$x^{i\prime} = x^i, \text{ (i=1,2,3)}, \; x^{4\prime} = x^4 + \left(x, \frac{v\prime}{c}\right). \tag{11.7b}$$

The metric follows from (11.1). We get

$$\begin{aligned} \eta_{ij} &= \delta_{ij} - \frac{v^{i\prime}}{c}\frac{v^{j\prime}}{c} && \text{(i; j= 1,2,3)} \\ &= -\frac{v^{i\prime}}{c} && \text{(1, 2, 3; j=4)} \\ &= -\frac{v^{j\prime}}{c} && \text{(i=1; j=1, 2, 3)} \\ &= -1. && \text{(i=j=4)} \end{aligned} \tag{11.8a}$$

with

$$(cd\tau)^2 = -\eta_{kl} dx^k dx^l. \tag{11.8b}$$

The inverse has the form

$$\begin{aligned} \eta^{ij} &= \delta^{ij} \quad (i; j = 1,2,3) \\ &= -\frac{v^{i\prime}}{c} \quad (i = 1,2,3;\ j = 4) \\ &= -\frac{v^{j\prime}}{c} \quad (i = 1;\ j = 1,2,3) \\ &= -\left(1 - \left|\frac{v'}{c}\right|^2\right). \quad (i = j = 4) \end{aligned} \tag{11.9}$$

Elementary calculations give the absolute value of light-velocity

$$|v_l| = c/\left(1 - \left|\frac{v\prime}{c}\right| cos\vartheta\right) \tag{11.10}$$

where ϑ denotes the angle between the vectors $v_l$of light-velocity and v'. Hence the light-velocity is anisotropic.

We consider the Michelson-Morley experiment. Let l be the length of the arms of the apparatus. Then, the total time for the travelling of the ray is

$$t = \frac{l}{c}\left\{\left(1 - \left|\frac{v\prime}{c}\right| cos\vartheta\right) + \left(1 - \left|\frac{v'}{c}\right| cos(180^0 - \vartheta)\right)\right\} = \frac{2l}{c}. \tag{11.11}$$

Therefore, the null-result of Michelson-Morley is received. The transformations (11.7) give the result of an event studied in the preferred frame Σ' for the same event as it would appear in the non-preferred frame Σ and vice versa.

We will now study the transformations in Σ which correspond to the Lorentz-transformations in Σ', i.e. they transform an event in Σ as it appears in Σ when it has the velocity w' measured in Σ'. We have the formulae (11.7) and for the moving object the same transformations hold, i.e.

$$\tilde{x}^{i'} = \tilde{x}^i \quad \text{(i=1,2,3)}, \quad \tilde{x}^{4'} = \tilde{x}^4 + \left(\tilde{x}, \frac{v\prime}{c}\right). \tag{11.12}$$

The transformations (11.7) and (11.12) yield from the transformations (11.4) by elementary computations the result

$$\tilde{x}^i = x^i + (\gamma - 1)\frac{(x,w\prime)}{|w\prime|^2}w^{i\prime} + \gamma x^4 \frac{w^{i}\prime}{c} + \gamma\left(x,\frac{v\prime}{c}\right)\frac{w^{i}\prime}{c} (1,2,3)$$

$$\tilde{x}^4 = \gamma\left(x^4 + \left(x,\frac{w'}{c}\right)\right) - \gamma\left(x^4 + \left(x,\frac{v'}{c}\right)\right)\left(\frac{w'}{c},\frac{v'}{c}\right) + (\gamma - 1)\left(\left(x,\frac{v'}{c}\right) - \frac{\left(x,\frac{w'}{c}\right)}{|w'|^2}(w',v')\right). \quad (11.13a)$$

The inverse formulae are

$$x^i = \tilde{x}^i + (\gamma - 1)\frac{(\tilde{x},w\prime)}{|w\prime|^2}w^{i\prime} - \gamma\tilde{x}^4\frac{w^{i}\prime}{c} - \gamma\left(\tilde{x},\frac{v\prime}{c}\right)\frac{w^{i}\prime}{c} \ (i=1,2,3)$$

$$x^4 = \gamma\left(\tilde{x}^4 - \left(\tilde{x},\frac{w'}{c}\right)\right) + \gamma\left(\tilde{x}^4 + \left(\tilde{x},\frac{v'}{c}\right)\right)\left(\frac{w'}{c},\frac{v'}{c}\right) + (\gamma - 1)\left(\left(\tilde{x},\frac{v'}{c}\right) - \frac{\left(\tilde{x},\frac{w'}{c}\right)}{|w'|^2}(w',v')\right). \quad (11.13b)$$

Any event computed in Σ at rest can be calculated inΣ when it moves with velocity w'.

The four-velocity in Σ is

$$\left(\frac{dx^i}{d\tau}\right) = \frac{dt}{d\tau}\left(\frac{dx^1}{dt},\frac{dx^2}{dt},\frac{dx^3}{dt}\right)$$

and in Σ′ the four-velocity is

$$\left(\frac{dx^{i}\prime}{d\tau\prime}\right) = \frac{dt\prime}{d\tau\prime}\left(\frac{dx^{1\prime}}{dt\prime},\frac{dx^{2\prime}}{dt\prime},\frac{dx^{3\prime}}{dt\prime}\right).$$

The last two relations give by the use of (11.7) and the standard transformations for the velocities in Σ and Σ′:

$$\frac{dx}{dt} = \frac{dx\prime}{dt\prime}\frac{1}{1-\left(\frac{1}{c}\frac{dx\prime}{dt'},\frac{v\prime}{c}\right)} \quad (11.14a)$$

$$\frac{dx'}{dt'} = \frac{dx}{dt}\frac{1}{1+\left(\frac{1}{c}\frac{dx}{dt},\frac{v'}{c}\right)}. \tag{11.14b}$$

In the special case that $\frac{dx^{i'}}{dt'} = v^{i'}$ we get

$$\frac{v}{c} = \frac{v'}{c}\frac{1}{1-\left|\frac{v'}{c}\right|^2}, \quad \frac{v'}{c} = \frac{v}{c}\frac{2}{1+\left(1+4\left|\frac{v}{c}\right|^2\right)^{1/2}}. \tag{11.15}$$

We will now give the transformation formulae for computing in the frame Σ an event which takes place in the frame Σ'. In the frame Σ the frame Σ' is described by the velocity $w' = v'$ in the formula (11.13), i.e.

$$\begin{aligned} \tilde{x}^i &= x^i + (\gamma-1)\frac{(x,v')}{|v'|^2}v^{i'} + \gamma x^4\frac{v^{i'}}{c} + \gamma\left(x,\frac{v'}{c}\right)\frac{v^{i'}}{c} \ (i=1,2,3) \\ \tilde{x}^4 &= \gamma^{-1}\left(x^4 + \left(x,\frac{v'}{c}\right)\right). \end{aligned} \tag{11.16}$$

The transformation law from Σ to Σ' is given by (11.7a) which implies

$$\begin{aligned} \tilde{x}^i &= x^{i'} + (\gamma-1)\frac{(x',v')}{|v'|^2}v^{i'} + \gamma x^{4'}\frac{v^{i'}}{c} \ (i=1,2,3) \\ \tilde{x}^4 &= \gamma^{-1}x^{4'}. \end{aligned} \tag{11.17a}$$

The inverse formulae are given by

$$\begin{aligned} x^{i'} &= \tilde{x}^i + (\gamma^{-1}-1)\frac{(\tilde{x},v')}{|v'|^2}v^{i'} - \gamma\tilde{x}^4\frac{v^{i'}}{c} \ (i=1,2,3) \\ x^{4'} &= \gamma\tilde{x}^4. \end{aligned} \tag{1.17b}$$

The formulae (11.17) are given at first by Tangherlini [*Tan* 61] and later on by Marinov [*Mar* 80]. Marinov stated the measurement of the velocity of the Earth of about $\left|\frac{v'}{c}\right| \approx 10^{-3}$ in agreement with the observed velocity relative to the CMB. Hence, we can identify the Earth with the non-preferred frame Σ and the CMB frame with the preferred frame Σ'.

All these results can be found in the article of Petry [*Pet* 86]. Furthermore, the paper contains in the non-preferred frame Σ the equations of Maxwell in a medium, the equations of motion of a point particle in the electro-magnetic field.

In addition, the experiments of Hook and Fizeau are studied being in agreement with the observed results. The Doppler-effect is also studied in the reference frame Σ. All these studies are omitted here.