Chapter 6

Split Plot Experimental Design

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Abstract

Spilt plot experimental design analysis is quite complicated because it involves the analysis of the main plot and then the sub plot. Due to its complicated nature, analysis of such a design is quite difficult for the understanding of students and most researchers. It is the appropriate design used for the analysis of a two factor experiment where all the treatments or factors cannot be contained in complete block design. This chapter explains how this design can be used and handled when analysing experiments.

Keywords

Split Plot, Design, Two Factor Experiment, Complete Block Design

6.1 Introduction

Every experimental design has its peculiar usefulness and cannot be relegated to the background when the situation calls for it. Whenever a researcher or an experimenter is faced handling two factor experiment which cannot be contained in a complete block design, then the split plot designs are used. The split plot design is thus suited for two factor experiment where the main factor is assigned to the main plot and the second factor assigned to the subplot which emanates from the division of the main plot. Thus the main plot becomes a block on its own and has subplot which holds treatment. The *main factor* is the factor which the researcher is very familiar with its characteristics. In a split plot design, the precision of measurement of effects of the main plot factors is sacrificed to improve that of the sub-plot factor. The relative size of the main effects and the precision of measurement of effects should not be the same for both factors – main factor and the sub factor. Assignment of factors to the main plot and the sub- plot is important and the guidelines to make this choice or assign the factors to the plot is determined by the relative size of the main effects and the precision of its measurement in relation to the researcher or experimenter's interest. As such it is often referred to as an experiment of convenience.

When the experimenter is considering greater precision for one factor as opposed to the other, the factor that requires a greater precision is assigned to the sub-plot and that with less precision assigned to the main plot. Taking for example a machine designer and a chemist considering a split plot design that involves the sizes of a machine type (S) and the chemical composition of food processed using these machines (C). The machine designer and the chemist are likely to assign these two factors as follows as per the precision required from each of on the measurement of the factors:

Machine Designer			
Factors	Factor type	Precision required	Plot type the factor must be assigned
Sizes of machines	Sub factor	More precision	sub plot -
Chemical composition of processed food	Main factor	Less precision	Main factor to be assigned to main plot

Chemist			
Factors	Factor type	Precision required	Plot type the factor must be assigned
Chemical composition of processed food	Sub factor	More precision	sub plot -
Sizes of machines	Main factor	Less precision	Main factor to be assigned to main plot

As regard the relative size of the main effects, the experimenter assigns factors to the main and subplot according to the relative size of their expected effects. For instance, if the main effect of one factor is larger and easier to detect than the other it is assigned to the main plot and the other assigned to the sub-plot. Taking for instance in an experiment to test the effects of different methods of processing food (P) and storing food (A) on nutrient loss; the factors in this experiment can be assigned in the design based on the relative size of their effects as shown below:

Assignment of factors based on the relative size of their effects			
Factors	Factor type	Size of effects factors	Plot type the factor must be assigned
Food processing methods (P)	Main factor	Larger effects	Main plot
Food storage methods (A)	Sub-factor	Lesser effects	Sub-plot

Another important factor in assigning factors is the management practices to be adopted in handling the factors under the design. For instance if the researcher is experimenting on the effect of a particular food item on the growth in terms of height on humans above twenty (20) year old (H) and humans below one (1) to twenty (20) years (G), since it has been established that humans cease to grow in height at age 21, those who are above twenty (H) needs to be assigned to the sub-plot while those between 1 and 20 be assigned to the main plot to minimize the effect of the food to those who already have the potential or capable of growing in height. This last factor applies to agricultural experiments. However with other industrial experiments, there are the hard-to-change factors which are assigned to the main plots and the easy-to-change factors which are assigned to the sub-plots. For instance assuming a sapele board is to be subjected to three different treatments (A, B, C) and then painted with three different paints (X, Y, Z) to ascertain its acceptability by users. These can be achieved in two ways: the first is to treat each sapele board in the three different conditions, divide each of them into three different portions and then paint them with the three selected paints; the other way is to divide each of the sapele board into three portions first, treat each of them and then paint them.

Therefore in a split plot experimental designs, the levels of the main plot factor multiplied by the levels of the sub-plot factor gives the number of treatments. It presupposes that if one is considering three levels of H (H_1, H_2, H_3) as sub-plot factor and G (G_1, G_2, G_3) as the main plot factor, then the number of treatments equals nine (9), 3 levels of H x 3 levels of G. if these are replicated for four (4) times, then there will be 36 treatments or observable units - 4 replicates x 3 levels of H x 3 levels of G.

To explain the step-by-step procedures used to analyse the split plot design, we can consider a hypothetical situation of designing an experiment involving three machines ($M_{1,}$ $M_{2,}$ M_3) made out of different materials such stainless stain, Aluminium and Iron respectively and their respective wear in terms of particles size and quantity (W_1, W_2, W_3) into the flour produced when used in milling the same quantity of maize.

In this particular case the main plot factors would be the machine types (M_1, M_2, M_3) and the subplot factors would be the different quantity of wear (W_1, W_2, W_3). Thus the split-plot design is shown below:

REPLICATION I		
M_1	M_2	M_3
W_1	W_1	W_1
W_2	W_2	W_2
W_3	W_3	W_3

REPLICATION II		
M_2	M_3	M_1
W_2	W_2	W_2
W_3	W_3	W_3
W_1	W_1	W_1

REPLICATION III		
M_3	M_1	M_2
W_3	W_3	W_3
W_1	W_1	W_1
W_2	W_2	W_2

In this experiment, the first replication shows how the split plot design looks like and randomization can be achieved as done in the other replications II and III. The design and the replications show how the main plots factors and the subplot factors would be arranged in the experiment. However when the experiment is conducted, the data obtained can be represented as follows:

REPLICATION I		
M_1	M_2	M_3
1.3	1.9	2.2
1.1	2.1	1.5
1.5	1.6	1.4

REPLICATION II		
M_2	M_3	M_1
1.2	2.2	1.9
1.4	1.5	2.1
1.3	1.4	1.6

REPLICATION III		
M_3	M_1	M_2
1.9	2.2	2.5
2.1	1.5	1.6
1.6	1.4	2.1

In order to do the calculation to complete the ANOVA table, the data obtained from the experiment is summarized and shown in the table below:

		W_1	**W_2**	**W_3**
REPLICATION I	M_1	1.3	2.1	1.5
	M_2	1.9	2.1	1.6
	M_3	2.2	1.5	1.4
REPLICATION II	M_2	1.3	1.2	1.4
	M_3	1.4	2.2	1.5
	M_1	1.6	1.9	2.1
REPLICATION III	M_3	2.1	1.6	1.9
	M_1	1.5	1.4	2.2
	M_2	1.6	2.1	2.5

6.2 Analysis of the Split Plot Design

For the analysis of the split plot design, one needs to understand that the size of the main plot is W times the size of the of the sub plot; the number of times the main plot treatment is tested is equal to the number of replications (r) used; there are three degrees of precision where the main plot factor is associated with the lowest degree of precision and the sub plot is associated with the highest degree of precision.

In doing the analysis for the split plot, the whole plot and the sub plot analyses must be done.

6.3 Whole Plot Analysis

The main plot treatments M_1, M_2 and M_3 within the blocks and handled as randomized complete block design. Once the main plot treatments are randomized as in the RCBD, no adjustment is required as regard the sum of squares for the main plot treatments (M).

In other to do the analysis the following steps should be followed:

		W_1	**W_2**	**W_3**
REPLICATION I	M_1	1.3	2.1	1.5
	M_2	1.9	2.1	1.6
	M_3	2.2	1.5	1.4
REPLICATION II	M_2	1.3	1.2	1.4
	M_3	1.4	2.2	1.5
	M_1	1.6	1.9	2.1
REPLICATION III	M_3	2.1	1.6	1.9
	M_1	1.5	1.4	2.2
	M_2	1.6	2.1	2.5

The first step is to calculate the replication total and the grand total.

In other to do these calculations the data has to be arranged in the tables below:

A cross tabulation of replication and machine type – main plot treatment (RM)

		W_1	**W_2**	**W_3**
REPLICATION I	M_1	1.3	2.1	1.5
	M_2	1.9	2.1	1.6
	M_3	2.2	1.5	1.4
REPLICATION II	M_2	1.3	1.2	1.4
	M_3	1.4	2.2	1.5
	M_1	1.6	1.9	2.1
REPLICATION III	M_3	2.1	1.6	1.9
	M_1	1.5	1.4	2.2
	M_2	1.6	2.1	2.5

Cross tabulation of Replication and Machine type (RM)

	M_1	**M_2**	**M_3**	**Rep Totals**
REPLICATION I	4.9	5.6	5.1	15.6
REPLICATION II	5.6	3.9	5.1	14.6
REPLICATION III	5.1	6.2	5.6	16.9
Machine type Totals	15.6	15.7	15.8	
Grand Total				47.1

Calculating the Grand Total (GT)

$$GT = \sum_{i=1,j=1}^{n=9} (4.9 + 5.6 + 5.1 + 5.6 + \cdots + 5.6) = 47.1$$

Calculating the Correction Factor (CF)

$$CF = \frac{(GT)^2}{rmw}$$

$$CF = \frac{(47.1)^2}{3 \ x \ 3 \ x \ 3}$$

$$CF = \frac{2218.41}{27}$$

$$CF = 82.16$$

where r=no. of replications, m=no. of levels of machine types, w=no. of levels of machine wears.

At this point one must compute the sum of squares of the main plot and these are done and illustrated below:

Calculating the Total Sum of Squares (TSS)

$$TSS = \sum sum\ of\ square\ of\ all\ observations - CF$$

$$TSS = \sum_{i=1,j=1,k=1}^{n=9} (m_1 w_1\, r_1)^2 + \ \ldots\ldots\ldots\ldots\ldots + (m_2 w_3 r_3)^2 - CF$$

$$TSS = \sum [(1.3)^2 + (2.1)^2 + (1.5)^2 + (2.2)^2 + \cdots + (2.5)^2] - 82.16$$

$$TSS = \sum [1.69 + 4.41 + 2.25 + 4.84 + \cdots + 6.25] - CF$$

$$TSS = 85.55 - 82.16 = 3.39$$

$$TSS = 3.39$$

Calculating the Replication Sum of Squares (RSS)

$$RSS = \sum \frac{[(R_{1_T})^2 + (R_{2_T})^2 + (R_{3_T})^2]}{mw} - CF$$

$$RSS = \sum \frac{[(15.6)^2 + (14.6)^2 + (16.9)^2]}{3\ x\ 3} - 82.16$$

$$RSS = \sum \frac{[\ 243.36 + 213.16 + 285.61]}{9} - 82.16$$

$$RSS = \sum (\frac{[\ 742.13]}{9} - 82.16)$$

$$RSS = 82.46 - 82.16$$

$$RSS = 0.30$$

Calculating the Machine Type Sum of Squares (MSS).

It should be noted here that the main plot factor is the machine type.

$$MSS = \sum \frac{[({M_1}_T)^2 + ({M_2}_T)^2 + ({M_3}_T)^2]}{rw} - CF$$

$$MSS = \sum \frac{[(15.6)^2 + (15.7)^2 + (15.8)^2]}{3\ x\ 3} - 82.16$$

$$MSS = \sum \frac{[\ 243.36 + 246.49 + 249.64]}{9} - 82.16$$

$$MSS = \sum (\frac{[\ 739.49]}{9} - 82.16)$$

$$MSS = 82.17 - 82.16$$

$$MSS = 0.01$$

Calculating the Replication and Machine Type Sum of Squares (RMSS).

Note: one is expected to use the cross tabulation table for replication and machine type

$$RMSS = \sum_{i=1,j=1}^{n=9} \frac{(m_1r_1)^2 + \ \dots\dots\dots\dots\dots + (m_2r_3)^2}{w} - CF$$

$$RMSS = \sum \frac{[(4.9)^2 + (5.6)^2 + (5.1)^2 + \cdots + (5.6)^2]}{w} - 82.16$$

$$RMSS = \sum \frac{[24.01 + 31.36 + 26.01 + \cdots + 31.36]}{3} - 82.16$$

$$RMSS = \sum \frac{[249.77\ ]}{3} - 82.16$$

$$RMSS = 83.26 - 82.16)$$

$$RMSS = 1.10$$

$$\text{Main plot Error Sum of Squares (MPESS)} = RMSS - RSS - MSS$$

$$MPESS = RMSS - RSS - MSS$$

$$MPESS = 1.10 - 0.30 - 0.01$$

$$MPESS = 0.79$$

6.4 Sub Plot Analysis

This is where the sum of squares of the sub plot factors are computed and these have been illustrated as below:

Cross tabulation of Machine type and machine wear (MW)

	M_1	M_2	M_3	W Totals
W_1	4.4	4.8	5.7	14.9
W_2	5.4	5.4	5.3	16.1
W_3	5.8	5.5	4.8	16.1

Calculating the Machine Wear Sum of Squares (WSS)

It should be noted that the machine wear is the sub plot factor for this particular experiment being considered for analysis.

$$WSS = \sum \frac{[(W_{1T})^2 + (W_{2T})^2 + (W_{3T})^2]}{rm} - CF$$

$$WSS = \sum \frac{[(14.9)^2 + (16.1)^2 + (16.1)^2]}{3\ x\ 3} - 82.16$$

$$WSS = \sum \frac{[\ 222.01 + 259.21 + 259.21]}{9} - 82.16$$

$$WSS = \sum (\frac{[\ 740.43]}{9} - 82.16)$$

$$WSS = 82.27 - 82.16$$

$$WSS = 0.11$$

Calculating the Sum of Squares of the interaction between Machine Type and Machine Wear

$$(M\ xW)SS = \sum_{i=1,j=1}^{n=9} \frac{(m_1w_1)^2 + \ \ldots\ldots\ldots\ldots\ldots + (m_3w_3)^2}{r} - \ CF$$

$$(MxW)SS = \sum \frac{[(4.4)^2 + (4.8)^2 + (5.7)^2 + \cdots + (4.8)^2]}{r} - \ 82.16$$

$$(MxW)SS = \sum \frac{[19.36 + 23.04 + 32.49 + \cdots + 23.04]}{3} - \ 82.16$$

$$(MxW)SS = \sum \frac{[248.23\ ]}{3} - \ 82.16$$

$$(MxW)SS = 82.74 - \ 82.16$$

$$(MxW)SS = 0.58$$

$$Sub\ Plot\ Sum\ of\ Squares\ (SPSS) = TSS - All\ Sums\ of\ Squares$$

$$Error\ Sub\ Plot\ Sum\ of\ Squares\ (ESPSS) = TSS - (RSS + MSS + RMSS + MPESS + WSS + (MxW)SS$$

$$SPSS = 3.39 - (0.30 + 0.01 + 1.10 + 0.79 + 0.11 + 0.58)$$

$$ESPSS = 3.39 - 2.89$$

$$SPSS = 0.50$$

6.5 Completing the ANOVA Table

Sources of Variation	df	Sum of Squares	Mean Sum of Squares	Fcal	Fcrit (5%)	Fcrit (1%)
Replication	`r -1 = 3 – 1 = 2	0.30	$\frac{0.30}{2} = 0.15$			
Main Plot factor (M)	m -1 = 3-1 = 2	0.01	$\frac{0.01}{2} = 0.005$	$\frac{0.005}{0.395} = 0.01$		
Error (M)	(r-1)(m-1) = 2 x 2 =4	0.79	$\frac{0.79}{4} = 0.395$			
Sub Plot factor (W)	w -1 = 3 -1 =2	0.11	$\frac{0.11}{2} = 0.055$	$\frac{0.055}{0.25} = 0.22$		
Interaction between (M x W)	(m-1)(w-1) = 2 x 2 =4	0.58	$\frac{0.58}{4} = 0.29$	$\frac{0.29}{0.25} = 1.16$		
Error (z)	m(r-1)(w-1) = 3 x 2 x 2 = 12	0.50	$\frac{0.50}{12} = 0.25$			
Total	rmw-1 = 27 -1 = 26					

Reading of the F-critical or tabulated values from the F-table at the various assigned levels of significance allowed.

For the case being considered, 1% and 5% levels of significance would be used.

Critical values of F for the 0.05 significance level:

	1	2	3	4	5	6
1	161.45	199.50	215.71	224.58	230.16	233.99
2	18.51	19.00	19.16	19.25	19.30	19.33
3	10.13	9.55	9.28	9.12	9.01	8.94
4	7.71	6.94	6.59	6.39	6.26	6.16
5	6.61	5.79	5.41	5.19	5.05	4.95
6	5.99	5.14	4.76	4.53	4.39	4.28
7	5.59	4.74	4.35	4.12	3.97	3.87
8	5.32	4.46	4.07	3.84	3.69	3.58
9	5.12	4.26	3.86	3.63	3.48	3.37
10	4.97	4.10	3.71	3.48	3.33	3.22
11	4.84	3.98	3.59	3.36	3.20	3.10
12	4.75	3.89	3.49	3.26	3.11	3.00
13	4.67	3.81	3.41	3.18	3.03	2.92
14	4.60	3.74	3.34	3.11	2.96	2.85

Critical values of F for the 0.01 significance level:

	1	2	3	4	5	6
1	4052.19	4999.52	5403.34	5624.62	5763.65	5858.97
2	98.50	99.00	99.17	99.25	99.30	99.33
3	34.12	30.82	29.46	28.71	28.24	27.91
4	21.20	18.00	16.69	15.98	15.52	15.21
5	16.26	13.27	12.06	11.39	10.97	10.67
6	13.75	10.93	9.78	9.15	8.75	8.47
7	12.25	9.55	8.45	7.85	7.46	7.19
8	11.26	8.65	7.59	7.01	6.63	6.37
9	10.56	8.02	6.99	6.42	6.06	5.80
10	10.04	7.56	6.55	5.99	5.64	5.39
11	9.65	7.21	6.22	5.67	5.32	5.07
12	9.33	6.93	5.95	5.41	5.06	4.82
13	9.07	6.70	5.74	5.21	4.86	4.62
14	8.86	6.52	5.56	5.04	4.70	4.46

Sources of Variations	Fcal	Fcrit (5%)	Fcrit (1%)
Main Plot Factor (M)	$\frac{0.005}{0.25} = 0.01$	df(2,4) = 6.94	df(2,4) = 18.00
Sub Plot Factor (W)	$\frac{0.055}{0.25} = 0.22$	df (2,12) = 3.89	df(2,12) = 6.93
Interactions (M x W)	$\frac{0.29}{0.25} = 1.16$	df (2,12) = 3.89	df(2,12) = 6.93

*Note in reading the f tabulated value for the main plot factor, the degree of freedom of the main plot factor is used against the degree of freedom of the error (M). However for the subplot factor and the interactions their respective degrees of freedom are used against the degree of freedom of error (z).

Taking the Decision and Making the Conclusion

Main Plot Factor (Machine Type):

$$Fcal\ (0.01) < Fcrit\ (\ df\ 2{,}4\ @\ 5\% = 6.94)$$

$$Fcal\ (0.01) < Fcrit\ (\ df\ 2{,}4\ @\ 1\% = 18.00)$$

Conclusion on Main Plot Factor (Machine Type):

There is no significance difference between the machine types used in the experiment at both levels of significance. This means the machine types have a similar effect.

Subplot Factor (Wear Type):

$$Fcal\ (0.22) < Fcrit\ (\ df\ 2{,}12\ @\ 5\% = 3.89)$$

$$Fcal\ (0.22) < Fcrit\ (\ df\ 2{,}12\ @\ 1\% = 6.93)$$

Conclusion on Subplot (Wear Type).

There is no significant difference between the various wear types. Meaning the wear types did not differ statistically

Interactions (M x W):

$$Fcal\ (1.16) < Fcrit\ (\ df\ 2{,}12\ @\ 5\% = 3.89)$$

$$Fcal\ (1.16) < Fcrit\ (\ df\ 2{,}12\ @\ 1\% = 6.93)$$

Conclusion on Interactions (M x W).

There exists no significance difference. Hence one can conclude that the interactions between the two did not differ statistically.

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