3

Determining the Optimal Stages Number of Module and the Heat Drop Distribution

3.1 Analytical Solutions

An important objective in the design of a multi-stage axial turbine is to determine the optimal number of stages in the module and the distribution of heat drop between stages.

Typically, a given quantity is the module's heat drop, and should vary the number of stages and the rotational speed (diameter). It should be understood that the circumferential velocity reduction, and hence the diameters of the stages, reduces the disc friction losses, increase height of the blades (and therefore reduce the proportion of end losses), decrease the flow path leakage. At the same time it leads to an increase in the optimal number of stages, which causes an increase in losses due to discs friction and an additional amount of the turbine rotor elongation. Immediately aggravated questions of reliability and durability (the critical number of revolutions), materials consumption, increase cost of turbine production and power plant construction.

A special place in the problem of the number of stages optimization is the correct assessment of the flow path shape influence, keeping its meridional disclosure in assessing losses in stages. As you know, the issue is most relevant for the powerful steam turbines LPC. It is therefore advisable for the problem of determining the optimal number of stages to be able to fix the form of the flow path for the LPC and at the same time to determine its optimal shape in the HPC and IPC.

It should also be noted that the choice of the degree of reaction at the stages mean radius (the amount of heat drop also associated with it) must be carried out with a view to ensuring a positive value thereof at the root. Formulated in this section methods and algorithms:

- may serve as a basis for further improvement of the mathematical model and complexity of the problem with the accumulation of experience, methods and computer programs used in the algorithm to optimize the flow of the axial turbine;
- allow the analysis of the influence of various factors on the optimal characteristics of the module, which gives reason for their widespread use in teaching purposes, the calculations for the understanding of the processes taking place in stages, to evaluate the impact of the various losses components on a stage operation;
- allow to perform heat drop distribution between stages and to determine the optimal number of stages in a module within the modernization of the turbine, i.e. at fixed rotational speeds (diameters) and a given flow path shape or at the specified law or the axial velocity component change along the cylinder under consideration.

A possible variant of the form setting of *n* stages group of the flow path can be carried out by taking the known axial and circumferential velocity components in all cross-sections, which the numbering will be carried out as shown in Fig. 3.1.

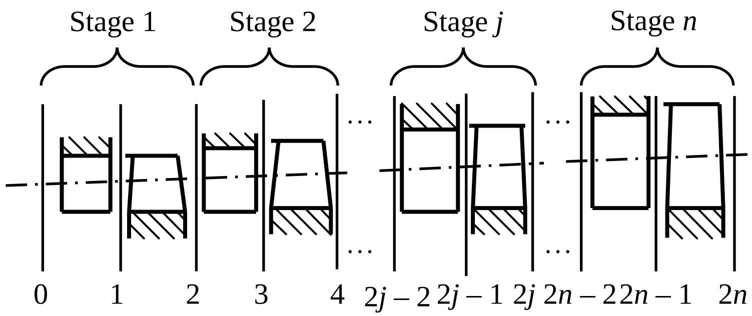

Figure 3.1 *The sections numbering in the turbine flow part section, consisting of n stages.*

The axial velocity components we refer to the axial velocity at the entrance to the stages group:

$$c_{jz} = K_{jz} c_{0z}, \quad \left(j = \overline{1, 2n} \right), \tag{3.1}$$

where K_{jz} – specified values.

The shape of the flow path center line determined by the introduction of coefficient

$$K_{jz} = u_j / u_0, \quad \left(j = \overline{1, 2n} \right), \tag{3.2}$$

By satisfying the conditions (3.1), (3.2) after optimization using the continuity equation $G = c_{0z}\rho_0 F_0 = c_{jz}\rho_i F_j$, $\left(j = \overline{1, 2n} \right)$ we can determine the shape of the flow path boundaries.

Assuming that we know the initial parameters of the working fluid at the turbine module inlet and the outlet pressure, i.e. theoretical heat drop in the group of n stags is known. Thermal process in the group of stages with the help of *hs*-diagram is shown in the Fig. 3.2.

Peripheral efficiency of the stages group determined by the formula

$$\eta_u = \frac{\sum_{j=1}^{n} L_u^{(j)}}{H_0} = \frac{\sum_{j=1}^{n} L_u^{(j)}}{i_0^* - i_{2TT,n}}$$

or taking into account (3.1) and (3.2) in a dimensionless form according to the expression

$$\eta_u = 2\nu_0^2 \overline{c}_{0z} \sum_{j=1}^{n} \left(K_{2j-1,u} K_{2j-1,z} \operatorname{ctg}\alpha_{2j-1} - K_{2j,u} K_{2j,z} \operatorname{ctg}\alpha_{2j} \right), \qquad (3.3)$$

where $\nu_0 = u_0 / C_0$; $C_0^2 = 2H_0$; $\overline{c}_{0z} = c_{0z} / u_0$.

We take into account the loss in the blades by applying velocity coefficients $\phi_j, \psi_j, \left(j = \overline{1, n} \right)$. Also, assume that the output of the intermediate stage a portion of the output energy may be lost. This fact will take into account by introducing a factor by which the output loss is defined as

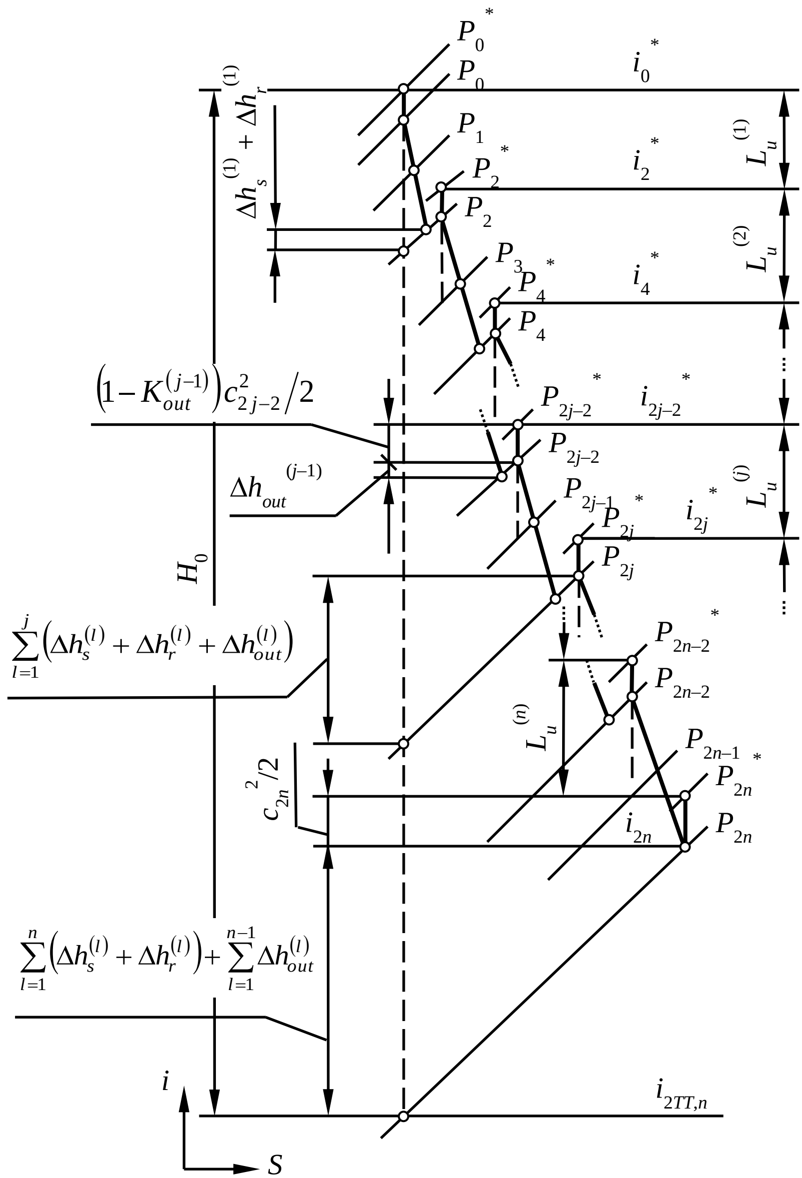

Figure 3.2 *The thermal process in the hs-diagram for the group of n stages.*

by introducing a factor by which the output loss is defined as

$$\Delta h_{out}^{(j)} = K_{out}^{(j)} \frac{c_{2j}^2}{2}, \quad \left(0 \le K_{out}^{(j)} \le 1; \quad j = \overline{1, n}\right), \tag{3.4}$$

Calculating losses in the guide vane and the rotor by formulas

$$\Delta h_s^{(j)} = \frac{1-\phi_j^2}{\phi_j^2} \frac{c_{2j-1}^2}{2}, \quad \Delta h_r^{(j)} = \frac{1-\psi_j^2}{\psi_j^2} \frac{w_{2j}^2}{2}, \quad \left(j = \overline{1, n}\right)$$

taking into account the factor of heat recovery α_n, the limit for the heat drop in the group of n stages can be written as:

$$A_3 = \left(1+\alpha_n\right) H_0 - \sum_{j=1}^{n} L_u^{(j)} - \sum_{j=1}^{n} \left(\Delta h_s^{(j)} + \Delta h_r^{(j)}\right) - \sum_{j=1}^{n-1} \Delta h_{out}^{(j)} - \frac{c_{2n}^2}{2} = 0. \tag{3.5}$$

Dividing equation (3.5) by u_0^2, taking into account (3.1), (3.2), as well as well-known kinematic correlations between velocity and flow angles after obvious transformations we obtain an expression for the limitation A_3 in the dimensionless form:

$$\begin{aligned} A_3 = {} & 2\bar{c}_{0z} \sum_{j=1}^{n} \left(K_{2j-1,u} K_{2j-1,z} \operatorname{ctg} \alpha_{2j-1} - K_{2j,u} K_{2j,z} \operatorname{ctg} \alpha_{2j}\right) + \\ & + \sum_{j=1}^{n} \left\{ \frac{1-\phi_j^2}{\phi_j^2} K_{2j-1,z}^2 \bar{c}_{0z}^2 \left(1+\operatorname{ctg}^2 \alpha_{2j-1}\right) + \frac{1-\psi_j^2}{\psi_j^2} \left[K_{2j,z}^2 \bar{c}_{0z}^2 \left(1+\operatorname{ctg}^2 \alpha_{2j}\right) - \right.\right. \\ & \left.\left. - 2K_{2j,u} K_{2j,z} \bar{c}_{0z} \operatorname{ctg} \alpha_{2j} + K_{2j,u}^2 \right] \right\} + \sum_{j=1}^{n-1} K_{out}^{(j)} K_{2j,z}^2 \bar{c}_{0z}^2 \left(1+\operatorname{ctg}^2 \alpha_{2j}\right) + \\ & + K_{2n,z}^2 \bar{c}_{oz}^2 \left(1+\operatorname{ctg}^2 \alpha_{2n}\right) - \frac{\left(1+\alpha_n\right)}{v_0^2} = 0. \end{aligned} \tag{3.6}$$

The task of definition of the angles α_j so that, given the parameters $v_0, \bar{c}_{0z}, K_{ju}, K_{jz}, \left(j = \overline{1, 2n}\right)$ and taken on the basis of some considerations (or defined by one of the possible methods), the quantities of velocity coefficients

$\phi_j, \psi_j, K_{out}^{(j)}, \left(j=\overline{1,n}\right)$ reaches its objective function (3.3) maximum and satisfies the constraint (3.6).

Mathematically the formulated problem reduces to finding:

$$\max_{\mathrm{ctg}\,\alpha_j} \eta_u = \frac{\sum_{j=1}^{n} L_u^{(j)}}{H_0}, \quad \left(j=\overline{1,2n}\right)$$

under the constraint (3.6).

Using the penalty functions method, you can reduce the problem of finding the extremum in the presence of constraints to the problem without limitation for the attached objective function

$$I^* = \eta_u - \Lambda A_3^2, \tag{3.7}$$

where Λ – penalty coefficient.

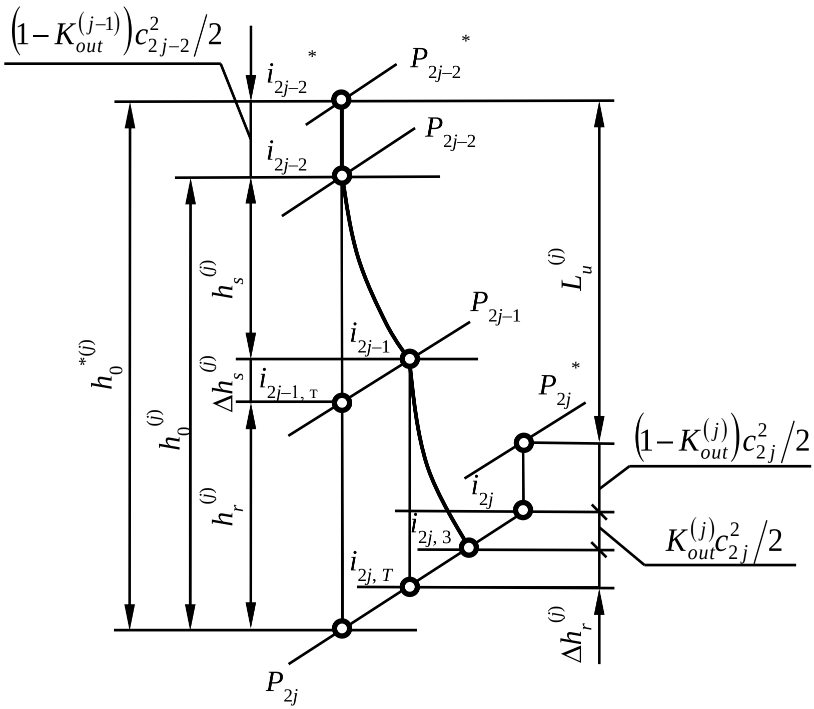

Figure 3.3 *The thermal process in hs-diagram for an intermediate j-th stage.*

Given the values of the velocity coefficients ϕ_j, ψ_j along the module, solution of the problem is simplified due to the possibility of its decision by indefinite Lagrange multipliers method. Differentiating the Lagrange function by variables $\operatorname{ctg}\alpha_j, \left(j=\overline{1, 2n}\right)$

$$\tilde{L}=\eta_u+\Lambda A_3, \tag{3.8}$$

where Λ – Lagrange multiplier, we find the following necessary optimality conditions

$$\frac{1}{\Lambda}=-\frac{1}{\nu_0^2}\left(1+\frac{1-\phi_j^2}{\phi_j^2}\frac{K_{2j-1,\,z}}{K_{2j-1,\,u}}\bar{c}_{0z}\operatorname{ctg}\alpha_{2j-1}\right), \quad \left(j=\overline{1, n}\right); \tag{3.9}$$

$$\frac{1}{\Lambda}=-\frac{1}{\nu_0^2}\left(\frac{1}{\psi_j^2}-\left(\frac{1-\psi_j^2}{\psi_j^2}+K_{out}^{(j)}\right)\frac{K_{2j,z}}{K_{2j,u}}\bar{c}_{0z}\operatorname{ctg}\alpha_{2j}\right),\quad \left(j=\overline{1,n-1}\right);\quad (3.10)$$

$$\frac{1}{\Lambda}=-\frac{1}{\nu_0^2\psi_n^2}\left(1-\frac{K_{2n,z}}{K_{2n,u}}\bar{c}_{0z}\operatorname{ctg}\alpha_{2n}\right).\quad (3.11)$$

Expressing all $\operatorname{ctg}\alpha_j\ (j\neq 2n)$ through $\operatorname{ctg}\alpha_{2n}$, and excluding Λ according to the third formula, we get:

$$\operatorname{ctg}\alpha_{2j-1}=\frac{1}{\mu_j}\frac{K_{2j-1,u}}{K_{2j-1,z}}\left(\frac{1-\psi_n^2}{\bar{c}_{0z}}-\frac{K_{2n,z}}{K_{2n,u}}\operatorname{ctg}\alpha_{2n}\right),\quad \left(j=\overline{1,n}\right);\quad (3.12)$$

$$\operatorname{ctg}\alpha_{2j}=\frac{1}{\chi_j+\psi_n^2K_{out}^{(j)}}\frac{K_{2j,u}}{K_{2j,z}}\left(\frac{\psi_n^2-\psi_j^2}{\psi_j^2\bar{c}_{0z}}+\frac{K_{2n,z}}{K_{2n,u}}\operatorname{ctg}\alpha_{2n}\right),\quad \left(j=\overline{1,n-1}\right),$$

Where $\mu_j=\psi_n^2\frac{1-\phi_j^2}{\phi_j^2}$; $\chi_j=\psi_n^2\frac{1-\psi_j^2}{\psi_j^2}$.

Substituting found in this manner $\operatorname{ctg}\alpha_{2j-1}$ and $\operatorname{ctg}\alpha_{2j}$ in the equation (3.6), we get the quadratic equation in the parameter $\operatorname{ctg}\alpha_{2n}^{opt}$

$$D\operatorname{ctg}^2\alpha_{2n}^{opt}+E\operatorname{ctg}\alpha_{2n}^{opt}+F=0,\quad (3.13)$$

where

$$D=\frac{K_{2n,z}^2}{K_{2n,u}}\frac{\bar{c}_{oz}^2}{\psi_n^2}\left(\sum_{j=1}^{n}\frac{K_{2j-1,u}^2}{\mu_j}+\sum_{j=1}^{n-1}\frac{K_{2j,u}^2}{\chi_j+\psi_n^2K_{out}^{(j)}}+K_{2n,u}^2\right);$$

$$E=-2\frac{K_{2n,z}}{K_{2n,u}}\frac{\bar{c}_{oz}}{\psi_n^2}\left(\sum_{j=1}^{n}\frac{K_{2j-1,u}^2}{\mu_j}+\sum_{j=1}^{n-1}\frac{K_{2j,u}^2}{\chi_j+\psi_n^2K_{out}^{(j)}}-K_{2n,u}^2\right);$$

$$F=\frac{c_{0z}^2}{\psi_n^2}\left[\sum_{j=1}^{n}\mu_j K_{2j-1,z}^2+\sum_{j=1}^{n-1}K_{2j,z}^2\left(\chi_j+\psi_n^2 K_{out}^{(j)}\right)+K_{2n,z}^2\right]+$$
$$+\sum_{j=1}^{n}\frac{1}{\mu_j}\left(\frac{1}{\psi_n^2}-\psi_n^2\right)K_{2j-1,u}^2-\sum_{j=1}^{n-1}\frac{K_{2j,u}^2}{\chi_j+\psi_n^2 K_{out}^{(j)}}\left(\frac{\psi_n^2}{\psi_j^4}-\frac{1}{\psi_n^2}\right)+$$
$$+\sum_{j=1}^{n}K_{2j,u}^2\frac{1-\psi_j^2}{\psi_j^2}-\frac{1+\alpha_n}{v_0^2}.$$

Using the solution of this equation, then define all the optimal angles $\operatorname{ctg}\alpha_j^{opt}\left(j=\overline{1,2n-1}\right)$ with (3.12), as well as optimal efficiency as a function of the set parameters with the help of (3.3).

Consider the important special case when $K_{j,u}=K_{j,z}=1$; $K_{out}^{(j)}=K_{out};\left(j=\overline{1,n-1}\right)$; $\phi_j=\phi,\psi_j=\psi,\left(j=\overline{1,n}\right)$. In this case, the formula (3.12) will have the form

$$\left.\begin{aligned}\operatorname{ctg}\alpha_{2j-1}&=\frac{1-\psi^2}{\mu\bar{c}_{0z}}-\frac{1}{\mu}\operatorname{ctg}\alpha_{2n}, \quad \left(j=\overline{1,n}\right);\\ \operatorname{ctg}\alpha_{2j}&=\frac{\operatorname{ctg}\alpha_{2n}}{1-\psi^2\left(1-K_{out}\right)}, \quad \left(j=\overline{1,n-1}\right),\end{aligned}\right\} \tag{3.14}$$

Where $\mu=\psi^2\dfrac{1-\phi^2}{\phi^2}$.

After the substitution of (3.14) into (3.6) we have a quadratic equation of the form (3.15) with the following values of the coefficients:

$$D=\frac{\bar{c}_{0z}^2}{\psi^2}\left[\frac{n}{\mu}+\frac{n-1}{1-\psi^2\left(1-K_{out}\right)}+1\right];$$

$$E=-2\frac{\bar{c}_{0z}}{\psi^2}\left[\frac{n}{\mu}+\frac{n-1}{1-\psi^2\left(1-K_{out}\right)}+1\right];$$

$$F=\frac{\bar{c}_{0z}}{\psi^2}\left\{1+n\mu+(n-1)\left[1-\psi^2\left(1-K_{out}\right)\right]\right\}+\frac{n}{\psi^2}\left[\frac{1-\psi^4}{\mu}+\left(1-\psi^2\right)\right]-\frac{1+\alpha_n}{\nu_0^2},$$

of which there is an optimal value $\operatorname{ctg}\alpha_{2n}$, then $\operatorname{ctg}\alpha_j\ \left(j=\overline{1,2n-1}\right)$ and from (3.13) the optimal efficiency

$$\eta_u=2\nu_0^2 n\left\{\frac{1-\psi^2}{\mu}-\bar{c}_{0z}\operatorname{ctg}\alpha_{2n}\left[\frac{1}{\mu}+\frac{n-\psi^2\left(1-K_{out}\right)}{n\left[1-\psi^2\left(1-K_{out}\right)\right]}\right]\right\}, \qquad (3.15)$$

optimal velocity ratios of the stages

$$\nu_j=\frac{u_j}{c_{0j}}=\left[2\bar{c}_{0z}\operatorname{ctg}\alpha_{2j-1}+\frac{1-\phi^2}{\phi^2}\bar{c}_{0z}^2\left(1+\operatorname{ctg}^2\alpha_{2j-1}\right)+\right.$$

$$\left.+\frac{\bar{c}_{0z}^2}{\psi^2}\left(1+\operatorname{ctg}^2\alpha_{2j}\right)-\frac{2\bar{c}_{0z}}{\psi^2}\operatorname{ctg}\alpha_{2j}+\frac{1-\psi^2}{\psi^2}\right]^{\frac{1}{2}}, \qquad (3.16)$$

optimal reactions

$$R=\frac{\phi^2-\bar{c}_{0z}^2\left(1+\operatorname{ctg}^2\alpha_{2j-1}\right)\nu_j^2}{\phi^2\left[1-\bar{c}_{0z}^2\left(1+\operatorname{ctg}^2\alpha_{2j-2}\right)\left(1-K_{out}\right)\nu_j\right]}. \qquad (3.17)$$

If adopted above conditions, we see that all the stages except the last, are the same. The final stage is different from all that is connected with the need to reduce the exit velocity loss that is completely lost at this stage $\left(K_{out}^{(n)}=1\right)$.

Using formulas (3.14)–(3.17) for values $\phi^2=0.96$, $\psi^2=0.9$ over a wide range $\bar{c}_{0z}$ from 0.2 to 1.0 and K_{out} from 0 to 1 the calculations were carried out, the results of which at values $\bar{c}_{0z}=0.4$ and $K_{out}=0.1$ are shown in Fig. 3.4.

Calculations have shown that for each value $\nu_0 = u/C_0$, i.e. heat drop for a given amount H_0 at a fixed circumferential speed *u*, an optimum number of stages exists at which the maximum efficiency of the module is reached.

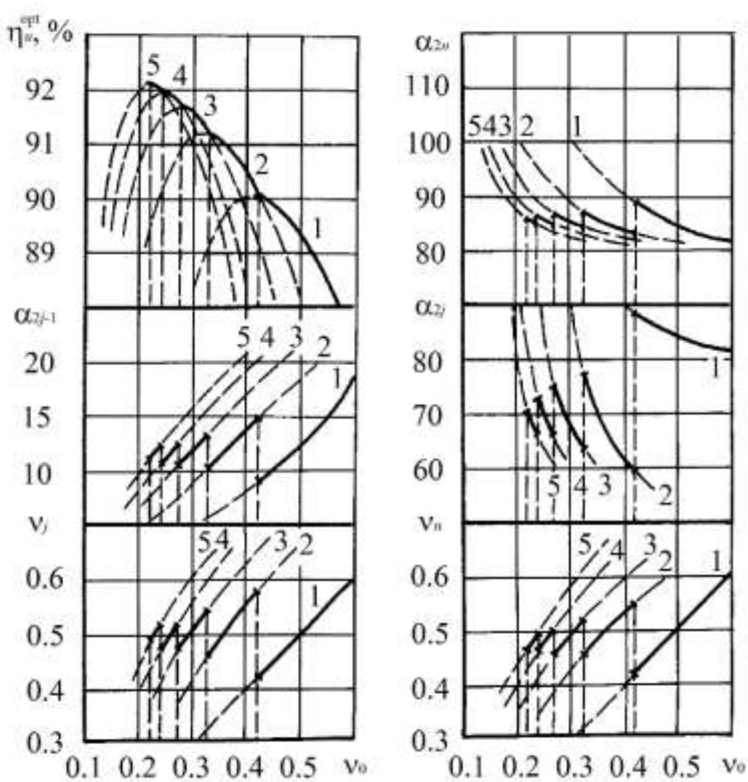

Figure 3.4 *The calculated optimal exit flow angles* α, *velocity ratios* ν *of intermediate and last stages, module efficiency* η *for different values of the heat drops* ($\bar{c}_{0z} = 0.4$, $\phi^2 = 0.96$, $\psi^2 = 0.9$, $K_{out} = 0.1$; $\left(j = \overline{1, n-1}\right)$)*. The numbers on the curves indicate the number of stages in the module. The bold line shows the envelope of the parameters, corresponding to the maximum efficiency.*

Assuming full utilization of the output velocity of the intermediate stages $\left(K_{out} = 0\right)$ the rotor exit angles of the intermediate stages α_{2j} $\left(j \neq n\right)$ can be very different from 90°.

The last stage flow exit angle α_{2n} in accordance with the calculation results must be done close to 90°, which corresponds with a minimum loss of output velocity. Angles downstream of the guide vanes lie in the range 10...17°, the optimum value of the velocity ratio in the range of 0.48...0.58. With increasing of number of stages in the module the range of acceptable changes of these values is narrowed.

In the case of output velocity loss in the intermediate stages $(K_{out} > 0)$ the picture somewhat changes. Increases the value of the heat drop, in which it is advisable to go to a larger number of stages, angles downstream of the intermediate stages α_{2j} are also close to 90°. There is a decrease in the velocity ratio values ν_j, the exit flow angles of the guide vanes α_{2j-1}, resulting in a slight drop in the optimum degree of reaction for the intermediate and for the last stages.

In the case of a *single stage*, assuming n = 1 peripheral stage efficiency is given by

$$\eta_u = \frac{L_u}{h_0} = \frac{u_1 c_{1u} - u_2 c_{2u}}{i_0^* - i_{2TT}}.$$

From (3.3) we obtain in the dimensionless form

$$\eta_u = 2\nu_{0z}^2 \bar{c}_{0z} \left(K_{1u} K_{1z} \operatorname{ctg} \alpha_1 - K_{2u} K_{2z} \operatorname{ctg} \alpha_2 \right). \tag{3.18}$$

For restrictions $\vec{A}_1$, $\vec{A}_2$ and $\vec{A}_3$ from equations (3.13), (3.6) is written (see notation on Fig. 3.1):

$$\left.\begin{aligned} A_1 &= i_0^* - i_1 - c_1^2/2 = 0; \\ A_2 &= i_0^* - i_2 - L_u - c_2^2/2 = 0; \\ A_3 &= i_0^* - i_{2TT} - L_u - \Delta h_s - \Delta h_r - c_2^2/2 = 0. \end{aligned}\right\}. \tag{3.19}$$

Drawing on the kinematic relations between velocities and flow angles, using the velocity triangles have

$$A_1 = 1 - \frac{i_1}{i_0^*} - \frac{k-1}{k+1}\lambda_0^2 K_{1z}^2 \frac{1+\mathrm{ctg}^2\alpha_1}{1+\mathrm{ctg}^2\alpha_0} = 0. \tag{3.20}$$

$$A_2 = 1 - \frac{i_2}{i_0^*} - \frac{k-1}{k+1}\lambda_0^2 K_{2z}^2 \frac{1+\mathrm{ctg}^2\alpha_2}{1+\mathrm{ctg}^2\alpha_0} -$$
$$-2\frac{k-1}{k+1}\frac{\lambda_0^2}{\bar{c}_{oz}\left(1+\mathrm{ctg}^2\alpha_0\right)}\left(K_{1u}K_{1z}\,\mathrm{ctg}\,\alpha_1 - K_{2u}K_{2z}\,\mathrm{ctg}\,\alpha_2\right) = 0. \tag{3.21}$$

$$A_3 = 2K_{1u}K_{1z}\bar{c}_{0z}\,\mathrm{ctg}\,\alpha_1 - 2\frac{K_{2u}K_{2z}}{\psi^2}\bar{c}_{0z}\,\mathrm{ctg}\,\alpha_2 +$$
$$+\frac{1-\phi^2}{\phi^2}\bar{c}_{0z}^2 K_{1Z}^2\left(1+\mathrm{ctg}^2\alpha_1\right) + \frac{K_{2Z}^2}{\psi^2}\bar{c}_{0z}^2\left(1+\mathrm{ctg}^2\alpha_2\right) +$$
$$+\frac{1-\psi^2}{\psi^2}K_{2u}^2 - \frac{1}{\nu_0^2} = 0. \tag{3.22}$$

Here for convenience introduced a dimensionless ratio

$$\lambda_0 = \frac{C_0}{a_*}, \tag{3.23}$$

where C_0 – stage inlet velocity defined by c_{0z} and α_0; $a_* = \sqrt{2\frac{k-1}{k+1}i_0^*}$ – velocity, equivalent to the critical value, for the ideal working fluid.

In the case of a perfect gas λ_0 is a reduced velocity at the stage inlet.

The stage optimization problem is solved using conjugate gradient method by maximizing the attached objective function $I^* = \eta_u - \Lambda A_3^2$, where Λ – penalty coefficient.

In the case of fixed velocity ratios φ and ψ from (3.12) we obtain the relationship between α_1^{opt} and α_2^{opt} analytically

$$\operatorname{ctg}\alpha_1^{opt} = \frac{1}{\mu}\frac{K_{1u}}{K_{1z}}\left(\frac{1-\psi^2}{\bar{c}_{0z}} - \frac{K_{2z}}{K_{2u}}\operatorname{ctg}\alpha_2^{opt}\right), \tag{3.24}$$

where denoted $\mu = \psi^2\dfrac{1-\phi^2}{\phi^2}$.

To determine the $\operatorname{ctg}\alpha_2^{opt}$ is needed to use the quadratic equation (3.13), which coefficients in the case of single stage are given by:

$$D = \frac{\bar{c}_{0z}^2 K_{2z}^2}{\psi^2}\left(1+\frac{1}{\mu}\frac{K_{1u}^2}{K_{2u}^2}\right);$$

$$E = -\frac{2K_{2u}K_{2z}\bar{c}_{0z}}{\psi^2}\left(1+\frac{1}{\mu}\frac{K_{1u}^2}{K_{2u}^2}\right);$$

$$F = \frac{1-\psi^4}{\psi^2}\frac{K_{1u}^2}{\mu} + \frac{\bar{c}_{0z}^2 K_{2z}^2}{\psi^2}\left(1+\mu\frac{K_{1z}^2}{K_{2z}^2}\right) + \frac{1-\psi^2}{\psi^2}K_{2u}^2 - \frac{1}{\nu_0^2}.$$

The reaction degree R of single stage is given by:

$$R = \frac{h_0 - \frac{1}{2}\left(\frac{c_1}{\phi}\right)^2}{h_0 - \frac{C_0^2}{2}} = \frac{1-\frac{K_{1z}}{\phi^2}\bar{c}_{0z}^2\nu_0^2\left(1+\operatorname{ctg}^2\alpha_1\right)}{1-\bar{c}_{0z}^2\nu_0^2\left(1+\operatorname{ctg}^2\alpha_0\right)}. \tag{3.25}$$

In the case where φ and ψ are functions of flow parameters, for a single stage the solution of the problem of determining the optimal parameters can be simplified by using the method of successive approximations:

1. Set the initial approximation φ, ψ and define the parameters for the stage using derived formulas.

2. The velocity coefficients are recalculated according to the obtained parameters and calculations are renewed from the item 1.

Calculations have shown that this process converges with high accuracy in a few iterations.

To investigate the influence of dimensionless parameters on the optimum stage performance computational study was conducted under various assumptions about the loss in the stage. The velocity coefficients were taken into account as a constant or dependent of the flow parameters. In the latter case, their determination was made using simplified dependency [28] with a bit increased losses on the rotor blades:

$$\left.\begin{aligned}\phi^2 &= 1 - 0.025\left[1 + \left(\frac{\varepsilon_\alpha}{90}\right)^2\right];\\ \psi^2 &= 1 - 0.040\left[1 + \left(\frac{\varepsilon_\beta}{90}\right)^2\right].\end{aligned}\right\}. \tag{3.26}$$

The increase in losses on the rotor blades in the presence of negative degree of reaction produced artificially by the formula

$$\psi^2 = \begin{cases}\psi^2, \text{ determined using (3.26), if } w_1 \le w_2;\\ \dfrac{\psi^2}{\psi^2 + \dfrac{w_1^2}{w_2^2}\left(1-\psi^2\right)}, \text{ if } w_1 > w_2.\end{cases} \tag{3.27}$$

The most complete calculations are made for the important special case when $K_{jz} = K_{ju} = 1 \quad (j = 1, 2)$.

Computational study has allowed to determine the optimal parametric dependencies of efficiency, angles α_1 and α_2, reaction *R*, velocity coefficients φ, ψ and loss factors ξ_s, ξ_r, ξ_{out} on $\bar{c}_{0z}$ and ν_0. The calculation results are shown in Fig. 3.5–3.7.

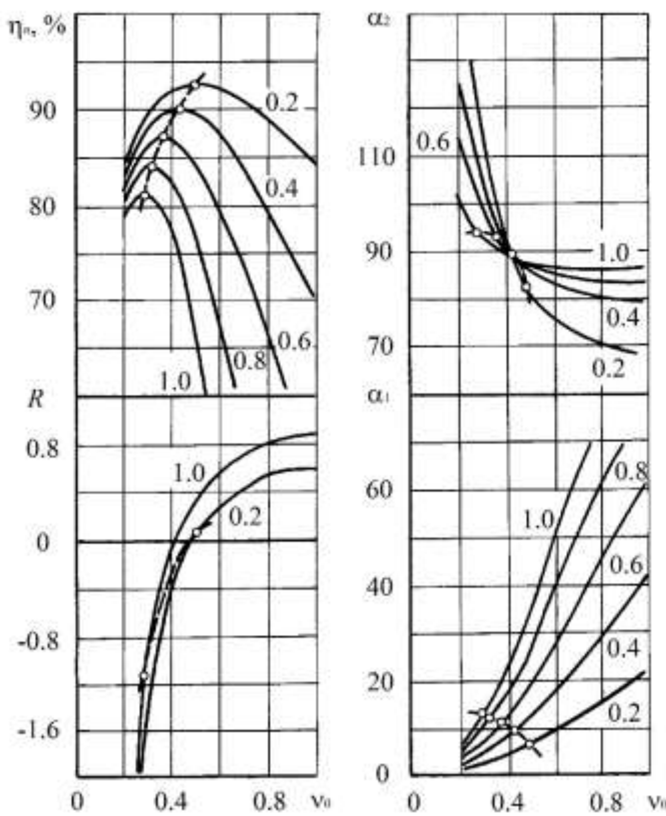

Figure 3.5 *Optimal characteristics of the turbine stage ($\phi^2 = 0.96$, $\psi^2 = 0.9$). The numbers refer to $\bar{c}_{0z}$ values. Circles mark the optimal parameters at optimal heat drop in the stage.*

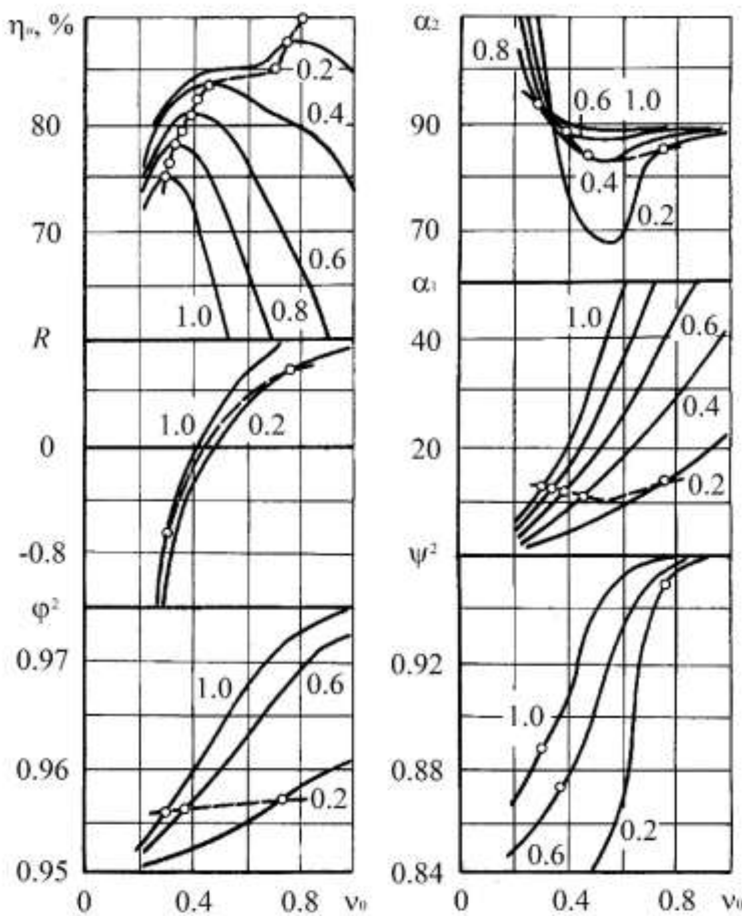

Figure 3.6 *Optimal characteristics of the turbine stage with velocity ratio, calculated by the formula (3.26). The numbers refer to $\bar{c}_{0z}$ values. Circles mark the optimal parameters at optimal heat drop in the stage.*

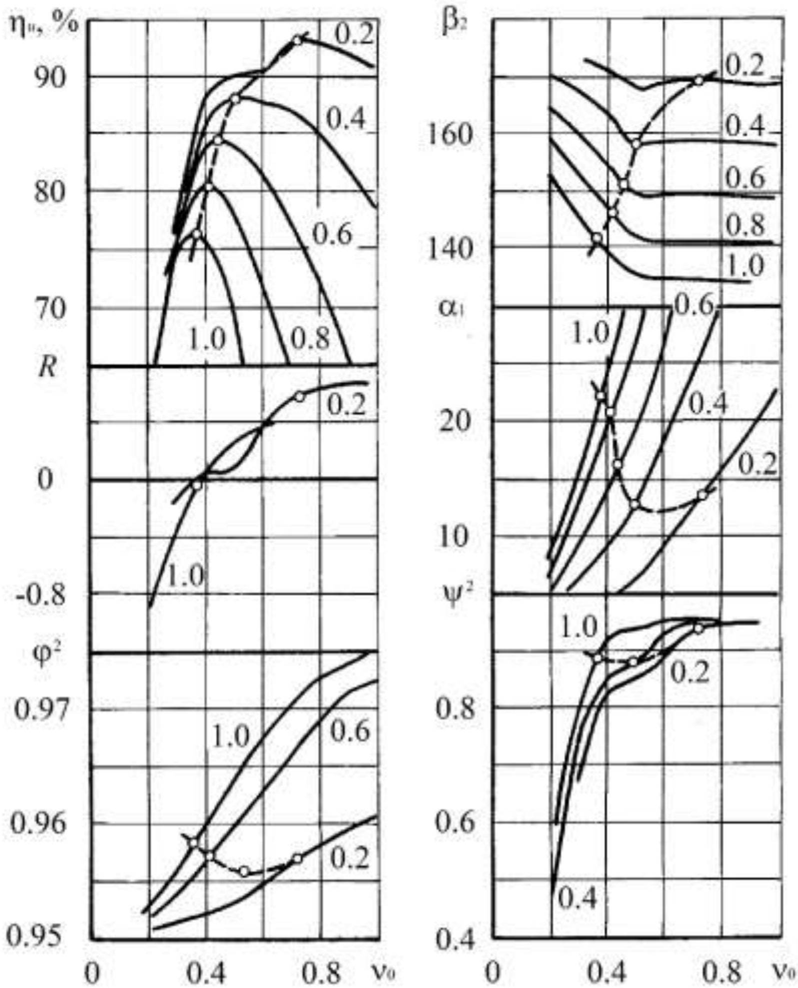

Figure 3.7 *Optimal characteristics of the turbine stage (φ, ψ calculated by the formula (3.26)) with ψ recalculated according to (3.27). The numbers refer to $\overline{c}_{0z}$ values. Circles mark the optimal parameters at optimal heat drop in the stage.*

3.2 Preliminary Design of the Multistage Axial Flow Turbine Method Description

In the early stages of the flow path (FP) design of the turbine, when determined the diameter, the blade heights, heat drops and other main characteristics of the stages, required to study alternatives with a view to the design solution, in the best sense of a quality criterion.

Most effectively, this problem is solved within the created turbine flow path CAD systems, because manage: to achieve a rational division of the designer, defining the strategy and computer, quickly and accurately perform complex calculations and presents the results in human readable numeric or graphical form; to take into account many different factors influencing the efficiency, reliability, manufacturability, cost and other indicators of the quality of the design being created; organize dialogue or fully automatic determination of optimal parameters, etc [29].

Most methods of the multi-stage turbine parameters optimization is designed to select the number of gas-dynamic and geometric parameters on the basis of the known prototype, the characteristics of which are taken as the initial approximation.

When using complex mathematical models, a large number of variables and constraints, the solution of such problems requires considerable computer time and for the purposes of CAD that require quick response of the system is often unacceptable.

It is desirable to have a method of design that combines simplicity, reliability and speed of obtaining results with an accuracy of the mathematical model, a large number of factors taken into account and optimized, the depth of finding the optimal variant. This inevitably certain assumptions, the most important of which are: the synthesis parameters of "good", competitive structure without attracting accurate calculation models; in-depth analysis and refinement of the parameters are not taken into account at the first stage; optimization of the basic parameters by repeatedly performing the steps of the synthesis and analysis.

Design of the FP in such a formulation will be called preliminary (PD). PD does not claim to such a detailed optimization of parameters, as in the above-mentioned methods of optimal design. Its goal – to offer a workable, effective enough design, the characteristics of which, if necessary, can be selected as the initial approximation for more accurate calculations.

Major challenges in creating a PD method are:

- a rational approach to the problem of the preliminary design, the selection of the quality criteria and the constraints system;
- development of a method for the multi-stage flow path basic parameters selection;
- formation of requirements for a mathematical models complex describing different aspects of turbines and their efficient numerical implementation;
- selection of the appropriate algorithm for finding the optimal solution;
- a flexible software creation for a dialog based solution of the design problems in various statements and visual representation of the results.

It is assumed that FP PD will be conducted immediately after the calculation of the turbine thermal cycle under known for each of the cylinders steam parameters i_0^*, P_0^* at the inlet, the backpressures for modules P_{2j}, mass flows G_j $\left(j=1,\ldots,n_{\mathrm{mod}}\right)$ and rotor frequency ω.

The task is selecting the number of stages in the modules n_j, root diameter D_{hj} and stages blades heights so as to achieve the maximum power of the cylinder, while ensuring reliability, manufacturability, or any other (material consumption, cost, size, etc.) pre-specified requirements.

The minimum acceptable reliability limits regulated (including safety factors) by static stresses in the blades and diaphragms, as well as detuning rotor blades of constant cross-section of the resonance. Technological constraints are reduced to a certain FP embodiment, task specific surface finish, as well as the use of standardized components – profiles, shanks, etc.

Common to powerful steam turbines HPC and IPC is the requirement of blading unification, when all stages are formed by trimming the top of the nozzle and rotor blades of the last stage of the module. At the same time it maintained a constant root diameter, angles α_{1h} and β_{2h}, as well as the root degree of reaction R_h at the uniform heat drops distribution between the stages and the constant axial velocity component in sections.

Consider ways of forming the cylinder FP, consisting of sections, satisfying, in particular, the above requirements of the unification. The idea of the method repeatedly expressed earlier. We apply it to the computer-aided FP design and make some modifications and generalizations.

3.2.1 Methods of the FP Synthesis

Consider one of the formulations of the PD problems, which we call the task I, in relation to the module.

Suppose that the root diameter D_h, root degree of reaction R_h and angle α_{1h} are known. The nozzle and rotor blades are considered to be twisted by law $c_u r = \text{const}$, which gives:

$$r_1 \operatorname{ctg} \alpha_1 = \text{const}; \quad r_2 \operatorname{tg} \beta_2 = \text{const}, \tag{3.28}$$

and to change the degree of reaction along the radius the relation is applicable

$$R_m = 1 - \frac{\bar{c}_z \dfrac{u}{C_0}}{\phi} \left[1 - \left(\frac{D_h}{D_h + l_1} \right)^2 \right] - \left(\frac{D_h}{D_h + l_1} \right)^2 (1 - R_h)$$

or the approximate formula [10]

$$R_m = 1 - \left(1 - R_h\right)\left(1 - \frac{l_1}{D_h + l_1}\right)^{1.8}. \tag{3.29}$$

First of all, the estimated process of steam expansion in the module is build. As we know neither the number nor the geometrical characteristics of the stages, it can be done only very approximately, evaluating the module efficiency η_{im}, such as by method [30]. This makes it possible to find the parameters of steam at the end of the actual process of expansion and, taking this process as linear, to evaluate the thermodynamic parameters at any pressure

$$P_{2n,\,\mathrm{mod}} \le P \le P_0^*.$$

To select the number of stages in the module let allow approximately uniform breakdown of the heat drops by the stages. Then, by setting the velocity ratio u/C_0 or evaluating its "optimal" (i.e. corresponding to the axial outlet flow from the stage – α_2 = 90) value, for example, by the formula [10]

$$\nu_0 = \frac{u}{C_0} = \frac{\phi\cos\alpha_1}{2\sqrt{1-R_m}} \quad \text{or} \quad \nu_h = \frac{u_h}{C_0} = \frac{\phi\cos\alpha_{1h}}{2\sqrt{1-R_h}}, \tag{3.30}$$

you can get

$$n = \frac{H_0 - \dfrac{c_{in}^2}{2}}{\dfrac{1}{8}\left(\dfrac{\omega D_h}{\nu_h}\right)^2 - \dfrac{c_{in}^2}{2}}, \tag{3.31}$$

where H_0 – module disposable heat drop; c_{in} – velocity at the inlet of the module; n – stage number – rounded up to the nearest integer.

Velocity c_{in}, which is equal to the axial component is determined by taking into account (3.30) according to the formula

$$c_{in} \approx c_{1z} = c_{1h} \sin \alpha_{1h} = \sqrt{2H_0 \left(1 - R_h\right) \sin^2 \alpha_{1h}} =$$
$$= \sqrt{\frac{4u_h^2 \left(1 - R_h\right)^2 \sin^2 \alpha_{1h}}{\phi^2 \cos^2 \alpha_{1h}}} = \omega \frac{D_h}{\phi} \left(1 - R_h\right) \operatorname{tg} \alpha_{1h}, \tag{3.32}$$

where φ is assumed equal to 0.96...0.98.

Introducing the notation

$$\Delta i = \frac{i_0^* - \dfrac{c_{in}^2}{2} - i_{2n_{\mathrm{mod}}}}{n}; \quad \Delta S = -\frac{S_0^* - S_{2n_{\mathrm{mod}}}}{n},$$

on the proposed steam expansion process for each of the stages the parameters in the gap between vanes without much error is determined based on the relationships:

$$P_{1j} = P\left(i_0^* - \frac{c_{in}^2}{2} - \left(j - 1\right) \Delta i + \frac{c_{1z}^2}{2} - \right.$$
$$\left. -\left(1 - R_{mj}\right) \left(\frac{\Delta i}{\eta_{im}} + \frac{c_{1z}^2}{2} \right), \quad S_0^* + \left(j - 1\right) \Delta S + \left(1 - R_{mj}\right) \Delta S \right), \tag{3.33}$$

$$i_{1j} = i\left(P_{1j}, S_0^* + \left(j + 1\right) \Delta S + \left(1 - R_{mj}\right) \Delta S\right), \tag{3.34}$$

$$\rho_{1j} = \rho\left(P_{1j}, i_{1j}\right), \quad j = 1, \ldots, n. \tag{3.35}$$

Pressures downward the stages are equal

$$P_{2j} = P\left(i_0^* - \frac{c_{in}^2}{2} - j\Delta i, \quad S_0^* + j\Delta S \right).$$

Nozzle vanes heights are determined from the continuity equation

$$G_j = \pi l_{1j} \left(D_h + l_{1j}\right) \rho_{1j} c_{1z}, \tag{3.36}$$

where c_{1z} taken from (3.32).

Solving (3.36) as a quadratic equation, we find

$$l_{1j} = \frac{1}{2}\left(-D_h + \sqrt{D_h^2 + \frac{4G_j}{\pi\rho_{1j}c_{1z}}}\right). \tag{3.37}$$

Since the value of R_{mj}, entering into (3.33)–(3.35), depends on the height of the blades, the iterative refinement of all the values determined by formulas (3.29), (3.33)–(3.35), (3.37) is needed. Taking as an initial approximation $R_h = 0$ typically achieve convergence of 2–4 iterations.

Instead of α_{1h} may be set, for example, the ratio D_m/l of the 1-st stage. We call this formulation as the problem II. In this case, immediately we find the height of the blades of the 1-st stage

$$l_1 = \frac{D_h}{\frac{D_m}{l} - 1}$$

and the 1-st stage degree of reaction at the mean radius using (3.29).

Angle α_1 of the 1-st stage is determined based on the continuity equation

$$G = \pi\rho_1 c_1 \sin\alpha_1 (D_m + l_1) l_1, \tag{3.38}$$

the obvious relation

$$c_1^2 = \phi^2 C_0^2 (1 - R_m) = \frac{\phi^2 \omega^2 (D_h + l_1)^2 (1 - R_m)}{4\nu^2} \tag{3.39}$$

and the conditions (3.30), what after simple calculations gives

$$\operatorname{tg}\alpha_1 = \frac{G}{\pi\rho_1 (D_h + l_1)^2 l_1 (1 - R_m)\omega}. \tag{3.40}$$

Furthermore, deleting C_0 from (3.39) using (3.30), we obtain

$$c_1 = \frac{\omega(D_h + l_1)(1 - R_m)}{\cos\alpha_1}; \quad c_{in} \approx c_1 \sin\alpha_1 . \tag{3.41}$$

The most advantageous number of stages in a module is determined by (3.31). Otherwise method II does not differ from the method I.

With the introduction of the coefficient

$$K_z = \frac{c_{1z,\,n}}{c_{in}}$$

methods I and II can be generalized to the case where the axial velocity components linearly vary from stage to stage. For this purpose, in the equations (3.33), (3.36) and (3.37) c_{1z} should be replaced by the value

$$c_{1z,\,j} = c_{in}\left[1 + \frac{(K_z - 1)(j - 1)}{n_m - 1}\right].$$

It should be borne in mind that when $K_z \neq 1$ the blade system unification condition ($\alpha_{1h} = \mathrm{const}$, $\beta_{2h} = \mathrm{const}$) is not satisfied.

Thus, as a result of solving PD problems in the statement I (II) certain basic characteristics of the FP are defined: the number of stages *n*, stages counter-pressures P_{2j}, root – level reaction degrees R_h, root diameter D_h, height of nozzle vanes l_{1j}, angles α_{1h} (ratio D_m/l of the 1-st stage).

3.2.2 Detailed Thermal Calculation

Next, to a more accurate assessment of the created design quality criteria and calculation of all required parameters is proposed to solve the inverse one-dimensional problem of thermal calculation of the FP for each of the stages of the cylinder. Known at this point data is not enough for this calculation.

Additionally, you need to specify the height of rotor blades, the geometric characteristics of the cascades, seals, etc.

Selection of missing values must be based on the design adopted, strength, technological and other requirements. For example, the height of the rotor blade can be obtained on the ground of information about the standard overlap or through the strict implementation of the conditions $\beta_{2h} = \text{const}$ in the group of stages. Using standard profiles data or generalized dependencies for the profile characteristics of arbitrary shape allows you to create a cascade, satisfying the requirements of efficiency, reliability and manufacturability. Selection of the main cascade parameters (a chord, stagger angle, pitch, etc.) it is advisable to carry out during the refinement of the velocity coefficients of crowns in the one-dimensional inverse problem of the FP thermal calculation. It’s quite a complicated independent problem which deserves special consideration.

The results of this calculation are the kinematic parameters of the flow in the gaps, the effective angles, cascade’s components of the kinetic energy loss and power parameters of stages. It also calculated the magnitude of stresses in the elements of design, weight, size and other characteristics. This information is sufficient to draw a conclusion about the quality of the built structure and the need to continue the design process.

A proximity in the selection of the basic FP parameters in the first PD stage compensated with the detailed account of the most factors affecting the quality parameters of the turbine in the model of thermal calculation. However, it should be borne in mind that during the synthesis of the FP should be set a number of parameters which are precisely determined only at the second calculation step. Therefore, there may be some differences in the parameters $u/C_0, \alpha_2, \eta_{i,\,\mathrm{mod}}$.

The most significant differences between the set and the refined α_{1h} value, which can reach 0.5...1.0° or more because of the stages number rounding to the nearest integer in the formula (3.31) and, as a consequence, the deflection u/C_0 in the formula (3.30) from the optimum. For this reason, and also because of methodologically inappropriate to set as the initial parameter, which subsequently must be determined (angle α_{1h}), the PD problem formulation II seems more rational.

3.2.3 Optimization

The desire to automate the PD process leads to the development of an algorithm for finding the optimal combination of the basic parameters of the flow path. With regard to the formulation II it is $D_h, R_h, \nu_h, D_m/l$ of the 1-st stage, and in case of failure of the unification – also K_z. The total number of variables to variable cylinder consisting of n_m modules thus does not exceed $5n_{\text{mod}}$.

They imposed restrictions

$$\left.\begin{array}{l} D_{h\min} \le D_h \le D_{h\max}; \\ R_{h\min} \le R_h \le R_{h\max}; \\ \nu_{h\min} \le \nu_h \le \nu_{h\max}; \\ (D/l)_{\min} \le (D/l) \le (D/l)_{\max}; \\ K_{z\min} \le K_z \le K_{z\max}. \end{array}\right\} \tag{3.42}$$

When selecting cascade's profiles during the detailed thermal calculation may be presented restrictions on static strength of the diaphragm and the rotor blades of the type

$$\sigma \le [\sigma], \tag{3.43}$$

design

$$\left.\begin{array}{l}\alpha_{1h} \geq \alpha_{1h\min};\\ \beta_{2h} \geq \beta_{2h\min};\\ n \geq n_{\max}\end{array}\right\} \tag{3.44}$$

and other.

To automatically design the FP, optimal in terms of the selected quality criteria, the designer must specify ranges of variable parameters and the required number of points in the search space defined by the conditions (3.42).

Sampling points generation is conducted using the LPτ sequences. Clarification of the optimal solution is achieved by reducing the ranges in the search. Typically, the amount of the search points ranges from a few dozen to several hundreds. Since the synthesis and thermal design of one point takes a few seconds, the maximum time to find the optimal variant is not more than a few minutes on a standard PC.