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Statement of the Axial Turbine's Flow Path Optimal Design Problem

The methodology of a turbine optimal design as a complex multi-level engineering system should support the operation with diverse mathematical models, providing for each design problem communication between the neighboring subsystems levels.

One approach to turbine design with using of block-hierarchical representation consists in the transition from the original mathematical models for the subsystems and numerical methods of optimization to "all-purpose" mathematical model and general method of parameters optimization.

1.1 Mathematical Models and the Object Design Problem

We will specify as original the mathematical model (OMM), which is a closed system of equations that describe the phenomena occurring in the designed object.

Regardless of the mathematical apparatus (algebraic, ordinary differential, integral, partial differential equations, etc.), OMM can be represented symbolically as follows:

$$\vec{Y} = \vec{Y}\left(\vec{B}, \vec{X}\right), \quad L\left(\vec{B}, \vec{X}\right) = 0, \tag{1.1}$$

where $\vec{X} = \left\{\vec{x}, \vec{u}\right\}$; $L\left(\vec{B}, \vec{X}\right)$ – the operator defining the model's system of equations.

The parameters $\vec{Y}$ characterize the quality indicators; $\vec{B}$ – entering the subsystems model from adjacent levels, the operational environment of the object. Parameters $\vec{X}$ can be either dependent, calculated by the OMM equation ($\vec{x}$) or independent, the choice of which provides the designer ($\vec{u}$). It is understood that the number of internal parameters of the object includes all internal parameters of the elements of underlying layers.

Significant simplification and unification of the subsystems description achieved by OMM approximation with a model, which we call a formal macromodel (FMM). We represent the FMM as a complete polynomial of the 2-nd degree, by which in many cases it is possible to approximate the output parameters with sufficient accuracy:

$$y(q) = A_0 + \sum_{i=1}^{n} (A_i + A_{ii} q_i) q_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ij} q_i q_j \ . \tag{1.2}$$

FMM parameters vector is expressed through the IMM parameters as

$$\vec{Q} = \vec{Q}(\vec{u}, \vec{B}). \tag{1.3}$$

hence FMM may be represented symbolically as follows:

$$y = y(\vec{B}, \vec{u}). \tag{1.4}$$

Comparing (1.4) and (1.1), we see that the FMM have no phase variables. The transformation of one model to another with a fewer number of variables or constraints, giving an approximated description of the investigated object or process compared to the initial, will be called *aggregation*. Thus, the FMM is aggregated with respect to (1.1).

The problems of the object's optimal design using models (1.1) and (1.4) will be called following:

$$\max_{\vec{u} \in U} Y_j(\vec{B}, \vec{x}, \vec{u}), \quad L(\vec{B}, \vec{X}) = 0; \tag{1.5}$$

$$\max_{\vec{u} \in U} y(\vec{B}, \vec{u}). \tag{1.6}$$

Suppose, that the problem (1.6) is solved for all possible values of the vector $\vec{B}$ that allows you to build approximation dependencies

$$y^{\text{opt}} = y^{\text{opt}}\left(\vec{B}\right), \quad \vec{u}^{\text{opt}} = \vec{u}^{\text{opt}}\left(\vec{B}\right), \tag{1.7}$$

containing information on all kinds of optimal designs. The model (1.7) is aggregated with respect to (1.4) and (1.5). The same could be made with the OMM: by virtue of solving the equations of the model would have disappeared phase, and by optimizing – control variables. Usually, however, this task is too complex for the numerical solution.

An approximate solution can be achieved with the help of disaggregation, i.e. mapping of aggregated variables in the model space of OMM. Substituting (1.7) to (1.1), we obtain:

$$Y_j = Y_j\left(\vec{B}, \vec{x}, \vec{u}^{\text{opt}}\left(\vec{B}\right)\right), \quad L\left(\vec{B}, \vec{X}\right) = 0,$$

where are $\vec{x}^{\text{opt}}$ and Y_j^{opt} – solution of OMM.

For example, in the optimal design of turbine cascade the quality criterion is the energy loss ratio, OMM – ideal gas motion and the boundary layer on the profile equations, phase variables – flow parameters, control – profile shape, cascade spacing and others.

In practice, instead of the loss calculation OMM various empirical loss calculation methods used, which, in fact, are FMMs of form (1.7), because does not take into account information about any and just about the currently best ("optimum") profile cascades. In this way, at higher design levels use only the information on the improved aerodynamic profiles loss ratio.

The approach described can be applied to multi-level design of complex systems.

1.2 Optimization of Complex Technical Devices

1.2.1 Design Hierarchy

Block-hierarchical representation of the design process, implemented with the creation of complex technical devices, leads to a problem of such complexity that can be effectively resolved by means of modern computing, and the results of the decision – understood and analyzed by experts. Typically, the design hierarchy of tasks is formed along functional lines for turbine can have the form shown in Fig. 1.1.

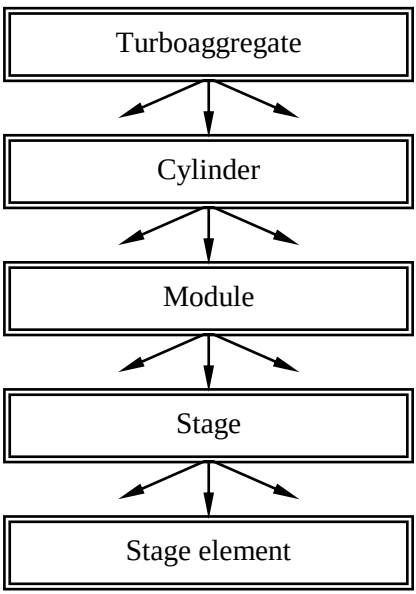


Figure 1.1 *Hierarchy of turbine design problems.*

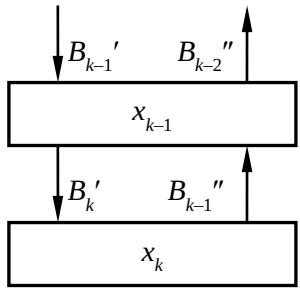


Figure 1.2 *Nearby hierarchy levels of optimization problems.*

The uniformity of mathematical models of the subsystems of the same level and local optimality criteria make it possible to organize the process of multi-level design, providing maximum global quality criterion of the whole system, in our case – the turbine. This process is based on the idea of so-called multilevel optimization approximation scheme that involves aggregation of mathematical models of the subsystems in the hierarchy when moving upward and disaggregation based on optimization results when moving downwards.

The problem of optimization the subsystem parameters described by OMM has the form (1.5). It can be solved by the methods of nonlinear programming and optimal control, depending on the form of the equations and the optimality criterion of the OMM.

Consider the solution order for the problems hierarchy of the system parameters optimization. Input parameters of *k*-level subsystem form of the set of internal and external parameters of the higher (*k*–1)-level subsystem. Feedback is carried out at the expense of the influence of the output parameters $\vec{B}_{k-1}''$ of the subsystem of *k*-level which with respect to the (*k*–1)-th subsystem is external. Complete vector of (*k*–1)-level external parameters, thus consists of a vector of external parameters $\vec{B}_{k-1}'$, coming from the higher-level and lower-level subsystems of vectors $\vec{B}_{k-1}''$ (Fig. 1.2).

Moving from the bottom up, we solve the problem of the form (1.6) at each *k*-level for all possible values of the vector of external parameters coming from a higher level. In this phase *k*-level variables are excluded from the internal parameters of the (*k*–1)-level model by effect of equations describing the *k*-level subsystem, and control – as a result of optimization. Thus, at each level above information is transmitted not about all, but only about the best projects of lower-level subsystems:

$$\vec{B}''_{k-1} = \vec{Y}_k^{\text{opt}}\left(\vec{B}'_k\right). \tag{1.8}$$

At the top, the 1-st level, from the problem (1.5) output parameters found, and predetermine external parameters of the level 2 subsystems, which makes it possible to restore the optimum parameters of the 2-nd level, solving the same problem (1.5). This disaggregation process extends to the lowest level, with the result that the optimal parameters are determined by all the subsystems that make up the complex technical systems.

To implement practically the described scheme is possible using FMM subsystems. In terms of the FMM problem (1.5) is written in a form similar to (1.6):

$$\max_{\vec{u}_k \in \vec{U}_k} y_k\left(\vec{u}_k, \vec{B}'_k\right), \tag{1.9}$$

which immediately follows

$$\vec{B}''_{k-1} = y_k^{\text{opt}}\left(\vec{B}'_k\right), \tag{1.10}$$

which is quite similar to (1.8), but has the advantage that it is a known polynomial of the form (1.2).

Methods based on the use of FMM is characterized in that before starting to solve the optimization problem on (*k*–1)-th level, it is replaced from the OMM

to FMM according to the condition (1.9). Driving multilevel optimization using FMM, is very flexible, allowing you to change the setting if necessary optimization tasks at any level due to changes in the components of vectors $\vec{Q}_k\left(\vec{u}_k, \vec{B}'_k\right)$.

1.2.2 A Numerical Method for the Implementation of the Multilevel Optimization Approximation Schemes

The current level of possibilities of computer technology and mathematics allow for a new approach to the organization of the block-hierarchical representation of the process of optimal design of axial turbine flow path (Fig. 1.1) and the information exchange between adjacent levels (Fig. 1.2). The essence of this approach lies in the application of the principle of recursion, provides automatic bypass facilities at all levels and solution for each object its local optimization problem in accordance with a predetermined scenario.

On the basis of this method created invariant subsystem of recursive object-oriented multi-criteria, multi-mode and multi-parameter optimization, providing solution of optimization problems, taking into account various types of parametric, structural, technological and functional limitations. Designed for its optimization techniques are universal, and the search for the optimal solution for each object is carried out in accordance with the scenarios of computing processes optimization.

Optimization scripts for all objects of all levels are formed and defined by set of components of the following vectors and lists:

- optX – address list parameters to be optimized;
- lXmin, lXmax – vectors defining the allowable range of variation of parameters to be optimized;
- lYcq – address list of the object settings and quality criteria;

- lYw – object quality criteria weight vector;
- lYfl – address list of parameters and functional limitations;
- flMin, flMin – functional limitations permissible change vectors;
- lYd – address list of settings – parametric constraints;
- dlMin, dlMin – parametric constraints permissible change vectors;
- lReg – list of regime (changing during the operation of the facility) parameters;
- sRegim – list of lines with the data on the values of operating parameters and the appropriate time of the object for these values;
- lLine – address list of parameters whose values are changed in the process of optimization by linear interpolation between the same type of parameters to be optimized nearby objects;
- optM – method for solving the optimization problem of the local object.

Forming all the lists, enumerated above, for all level objects and calling a recursive function, which includes a set of corresponding optimization algorithms, an automatic objects bypass and solving optimization problems for each of them is carried out.

1.3 Building Subsystems FMM

1.3.1 FMM Basics

As noted, the FMM is an approximation of the original model, which means it can be obtained by statistical processing of the results of numerical experiment using OMM. The complexity of solving the equations of the original model forces minimize the number of sampling points, which is practically achieved by using methods of the theory of experiment design. Get the response function in the form (1.2) can, in particular, on the basis of

three-level Box and Benken plans [1]. Special selection of sampling points on the boundary of the approximation

$$-1 \le q_l \le 1, \quad l = 1, \ldots, N \tag{1.11}$$

and in its center possible in accordance with the least squares method to obtain the values of the coefficients according to (1.2), without resorting to the numerical solution of the normal equations. The number of sampling points is in the range from 13 at N = 3 to 385 at N = 16.

Similarly, relations (1.2) can also be obtained by using the three-level saturated plans by Rehtshafner [2]. In this case, the dimension of the observation vector will vary from 16 at N = 4 to 232 at N = 20. The feature of these plans is that it is the most economical plans that require a minimum number of calculations to generate a vector of observations, i.e. the number of calculations (experiments) equal to the number of the coefficients according to (1.2).

When creating subsystems FMM quality criteria, should be noted, that at lower levels increases the degree of detailed description of the design objects, which leads to an increase in the dimension of $\vec{Q}_k$ vectors. If the dimension exceeds the permissible (N = 20), or for any reason is limited, for example, due to the complexity of OMM, it can be reduced by replacing a number of components of the control parameters vector defined by the laws of their change, by numbers of the same type subsystems (objects) at the considered design level. For example, in the formal macromodelling of the multi-stage turbine flow path efficiency, may be appropriate to change the degree of reaction, disposable heat drop and so forth linearly from stage to stage. To ensure information consistency between FMMs of adjacent levels, in a number of components of the vector $\vec{Q}_{k+1}$ should be required to include parameters that uniquely determine the position of the subsystems in the settings space of a higher k-level.

It should be noted that in addressing the increasingly complex, multi-parameter, multi-mode and multi-criteria problems of optimal design increases the likelihood of multimodal objective functions.

Using the dependency of the form (1.2) for the approximation of the objective functions and functional limitations in this case can lead to a decrease in the accuracy and adequacy of the obtained with its help optimal solutions for the projected objects or subsystems.

1.3.2 The Method of Improving the FMM Accuracy

The analysis of the structure of formula (1.2) shows, that its second term is a superposition of the parabola from each independent parameter that mainly determines the failure of functions of the form $\sum_{i=1}^{n}\left(A_i q_i + A_{ii} q_i^2\right)$ take into account the more complex nature of real dependencies, having, for example, bends and local extremes. We will use a second member according to (1.2) to reflect the independent effect of the parameters on the approximated function, and replace it with a more perfect form of addiction.

It is obvious that in the general case, the shape and structure of dependency, reflecting the influence of each parameter, is unique. Given that a priori a kind of dependency is not known, to solve this problem and ensure that the principle of universality, the second term of the form $\sum_{i=1}^{n}\left(A_i q_i + A_{ii} q_i^2\right)$ should be replaced with the superposition of interpolation cubic splines. As known, the interpolation cubic splines allow with a high degree of accuracy and adequacy to describe features of varying complexity, including multi-extremal. Thus, taking into account this replacement, the formal macromodel of the form (1.2) will be as follows:

$$y(q) = A_0 + \sum_{i=1}^{n}\left(a_{ij} + \left(b_{ij} + \left(\frac{c_{ij}}{2} + \Delta q_{ij}\cdot\frac{d_{ij}}{6}\right)\Delta q_{ij}\right)\Delta q_{ij}\right) + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{ij} q_i q_j , \quad (1.12)$$

where $a_{ij}, b_{ij}, c_{ij}, d_{ij}$ – cubic spline coefficients of current (j-th) interpolation section of the i-th independent variable. For each independent normalized variable q_i there are several areas in the interpolation range between –1 and +1; Δq_{ij} – the distance between the current value q_i and coordinate of the initial node of j-th section of the spline, which q_i coordinate value is between the initial coordinates of (j-th) and final (j + 1-th) of its nodes.

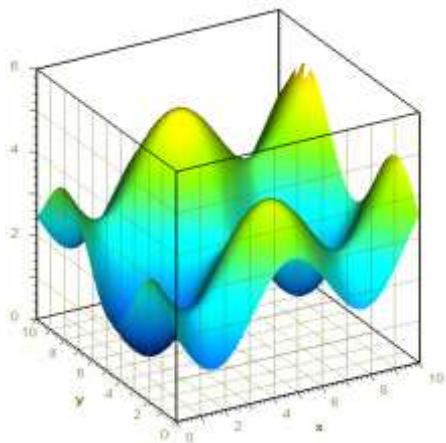
Of course, for the coefficients $a_{ij}, b_{ij}, c_{ij}, d_{ij}$ of dependence (1.12) determination additional computational experiment is needed. This experiment carried out at the points of a normed space of independent variables q_i. The length of the interpolation areas and their nodes coordinates are the same for all the independent variables. The number of sections is given. The minimum required number of sections is four. In this case, an additional calculation of the objective functions at four points (1; 0.5; 0.5; 1) by each variable q_i is needed.

To ensure the principle of an independent effect of each variable, other variables in the calculation are assigned by the value 0 ($q_j = 0$), which corresponds to the center of the accepted range of their changes. It should also be noted that in the case of Rehtshafner's design plans to create more accurate FMM of form (1.12) the number of computations by OMM is reduced for each independent parameter of FMM by two and equal, accordingly, two, since the other two points coincide with the points of the Rehtshafner's plan and their corresponding calculations for OMM performed at the stage of creating a FMM

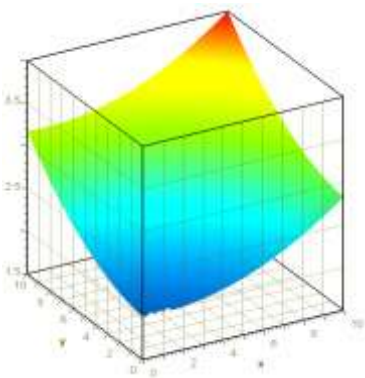
of form (1.2). For clarity, in Fig. 1.3 shows a comparison of the accuracy and the adequacy of the approximation of test functions of the form:

$$Z = 2 + 0.1X^2 + 0.1Y^2 - \sin X - \sin Y, \tag{1.13}$$

by formal macromodel of the form (1.2) and the form (1.12).



a



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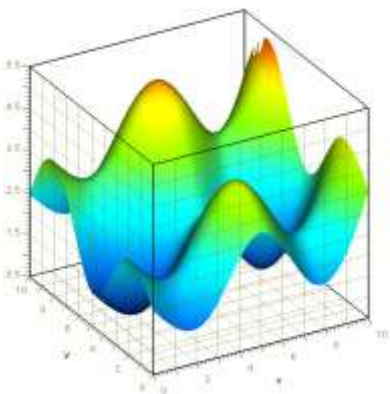

c

Figure 1.3 *Comparison of the accuracy of approximation of multimodal function using formal macromodel: a – test multimodal function of the form (1.13); b – approximation of functions of the form (1.13) using formal macromodel of the form (1.2); c – approximation of functions of the form (1.13) using formal macromodel of the form (1.12).*

1.4 Optimization Methods

1.4.1 General Information About the Extremal Problems

To solve problems with the single criterion of optimality rigorous mathematical methods are developed.

Direct methods of the calculus of variations – one of the branches of the theory of extreme problems for functional – reduce the problem of finding the functional extremum to the optimization of functions.

There are analytical and numerical methods for finding optimal solutions. As a rule, the real problems are solved numerically, and only in some cases it is possible to obtain an analytical solution.

Functions optimization using differentiation

Finding the extremum of the function of one or more variables possible by means of differential calculus methods. It's said that the $\hat{x}$ point gives to function $f(x)$ local maximum, if there is a number $\varepsilon > 0$ at which from the inequality $|x - \hat{x}| < \varepsilon$ the inequality $f(x) \le f(\hat{x})$ comes after.

The function is called one-extremal (unimodal) if it has a single extremum and multi-extremal (multimodal), if it has more than one extremum. The point at which the function has a maximum or minimum value of all local extrema, called a point of the global extremum.

A necessary condition for an extremum of a differentiable function of one variable gives the famous Fermat's theorem: let $f(x)$ – function of one variable, differentiable at the point $\hat{x}$. If $\hat{x}$ – local extreme point, then $f'(\hat{x}) = 0$.

The points at which this relationship is satisfied, called stationary. The stationary points are not necessarily the point of extreme. Sufficient conditions for the maximum and minimum functions of one variable – respectively $f''(\hat{x})<0$, $f''(\hat{x})>0$.

Before proceeding to the necessary and sufficient conditions for extrema of functions of several variables, we introduce some definitions.

The gradient of function $f(x)$ is a vector

$$\nabla f(x)=\begin{bmatrix}\dfrac{\partial f(x)}{\partial x_1}\\ \cdots\\ \cdots\\ \dfrac{\partial f(x)}{\partial x_n}\end{bmatrix},$$

$\nabla^T f(x)$ denotes the row vector

$$\nabla^T f(x)=\left\{\frac{\partial f(x)}{\partial x_1},\ldots,\frac{\partial f(x)}{\partial x_n}\right\}.$$

A square matrix of second derivatives

$$\nabla^2 f(x)=h(x)=\begin{bmatrix}\dfrac{\partial^2 f(x)}{\partial x_1^2} & \cdots & \dfrac{\partial^2 f(x)}{\partial x_1\partial x_n}\\ \cdots & \cdots & \cdots\\ \dfrac{\partial^2 f(x)}{\partial x_n\partial x_1} & \cdots & \dfrac{\partial^2 f(x)}{\partial x_2^n}\end{bmatrix}$$

is called the Hesse matrix or Hessian of function $f(x)$.

The real symmetric matrix H is called positive (negative) defined if $x^T = Hx > 0 (< 0)$ for every set of real numbers $x_1, x_2, \ldots, x_n$, not all of which are zero.

The necessary conditions for that $\hat{x}$ – the point of local extremum of n variables function $f(x)$, $x \in E^n$ are as follows:

1) the function $f(x)$ is differentiable in $\hat{x}$;

2) $\nabla f(x) = 0$, that is $\hat{x}$ – the stationary point; sufficient conditions for that $\hat{x}$ – local extreme point, but "1", "2" include the following;

3) Hessian is positive (negative) determined at the minimum (maximum), i.e. $\hat{x}^T H\hat{x} > 0 (< 0)$.

If the Hessian is positive (negative) defined for all $x \in E^n$, it is a sufficient condition of unimodality of the function. To test matrix A definiteness, Sylvester criterion is applied, according to which the necessary and sufficient condition for positive certainty are the inequalities:

$$a_{11} > 0, \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0, \ldots, \begin{vmatrix} a_{11} & \ldots & a_{1n} \\ \ldots & \ldots & \ldots \\ a_{n1} & \ldots & a_{nn} \end{vmatrix} > 0,$$

as to the negative certainty

$$-a_{11} > 0, \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0, \ldots, (-1)^n \begin{vmatrix} a_{11} & \ldots & a_{1n} \\ \ldots & \ldots & \ldots \\ a_{n1} & \ldots & a_{nn} \end{vmatrix} > 0.$$

Tasks for conditional extremum of function

This case involves determining the extremum in an infinite change range of variables $x_1, x_2, \ldots, x_n$. If optimized function imposed additional conditions (restrictions), talk about the problem of conditional extremum. In general, you want to find extremum $f(x)$, $x \in E^n$ under the constraints

$$\left.\begin{aligned} h_j(x) = 0, \quad j = 1, \ldots, m; \\ g_j(x) \geq 0, \quad j = 1, \ldots, p. \end{aligned}\right\} \tag{1.14}$$

To solve the problem (1.14) only with restrictions in the form of equations a method of Lagrange multipliers is used, which is based on the conduct of the Lagrange's function $L(x, \lambda) = f(x) + \sum_{j=1}^{m} \lambda_j h_j(x)$, where λ_j – undetermined Lagrange multipliers. We write the necessary conditions for optimality in the problem of conditional extremum with equality constraints

$$\left.\begin{aligned} \frac{\partial L}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{j=1}^{m} \lambda_i \frac{\partial h_j}{\partial x_i} = 0, \quad i = 1, \ldots, n; \\ \frac{\partial L}{\partial \lambda_i} = h_j(x) = 0, \quad j = 1, \ldots, m. \end{aligned}\right\} \tag{1.15}$$

It is a system of $n + m$ equations from which can be determined x_i, $i = 1, \ldots, n$, λ_j, $j = 1, \ldots, m$. A rigorous proof of the Lagrange conditions set out in the specific manuals. Explain the meaning of the method as follows. On the one hand, for all of x which satisfy the constraints $h_j(x) = 0$, $j = 1, \ldots, m$, obviously $L = (x, \lambda) = f(x)$.

On the other hand, the extreme point of the Lagrange function also satisfies these conditions (the second equation (1.14), and therefore, finding an extremum $L(x, \lambda)$, we simultaneously obtain a conditional $f(x)$ extremum. To

address the issue of the presence of a stationary point to be a local extremum in the problem of conditional extremum, let us expand Lagrange function in a Taylor series with a subject to the satisfaction of relations $h_j(x)=0$.

$$f(\bar{x}+\xi)-f(\bar{x})=L\left(\bar{x}+\xi,\hat{\lambda}\right)-L\left(\bar{x},\hat{\lambda}\right)=$$
$$=\sum_{i=1}^{n}\frac{\partial L}{\partial x_i}\xi_i+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial^2 L}{\partial x_i\partial x_j}\xi_i\xi_j+o(\|\xi\|^2), \quad (1.16)$$

and according to (1.15) the first term on the right side is zero. The expansion of a Taylor series $h_j(x)$ in the neighborhood of a stationary point $\bar{x}$ yields

$$\sum_{i=1}^{n}\frac{\partial h_j}{\partial x_i}\xi_i+o(\|\xi\|^2)=0,\quad j=1,\ldots,m, \quad (1.17)$$

Neglecting terms of higher order, write (1.16), (1.17) in the form

$$f(\bar{x}+\xi)-f(\bar{x})=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial^2 L}{\partial x_i\partial x_j}\xi_i\xi_j;\ \sum_{i=1}^{n}\frac{\partial h_j}{\partial x_i}\xi_i=0,\ j=1,\ldots,m. \quad (1.18)$$

If from the second equation (1.18) the dependent variables ξ_i, $i=1,\ldots,m$, can be expressed through independent ξ_k, $k=m+1,\ldots,n$, then substituting them in the first equation (1.18), we obtain a quadratic form relatively independent increments $\xi_{m+1},\ldots,\xi_n$. The stationary point $\bar{x}$ is a local conditional minimum (maximum), only if it is positive (negative) defined.

Optimization with constraints in the form of inequalities

Classical methods of finding the conditional and unconditional extrema of functions discussed above, in some cases, allow to solve problems with inequality constraints.

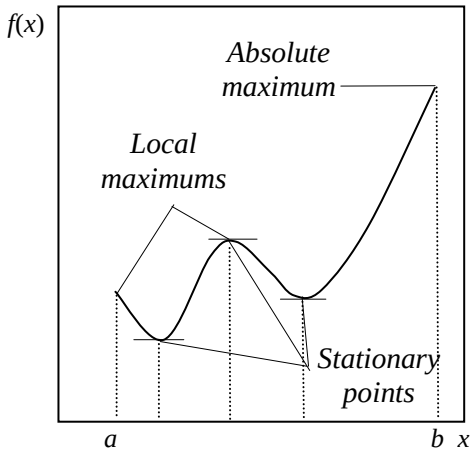

Figure 1.4 *For extremum determination of the functions of one variable in the interval.*

Let the task of finding the maximum of a function of one variable $f(x)$ on the interval $a \le x \le b$. Using the necessary optimality conditions, we find the roots of $f'(x)=0$ which lie in the interval $[a,b]$; We check the suffi-ceent conditions for maximum $f''(\hat{x})<0$ and choose the points corresponding to the maximum. Also, we compute the function values at the borders of a segment, where it can take higher values than the interval (Fig. 1.4).

We turn now to the case of several variables and consider the optimization problem: find a maximum $f(x)$, $x \in E^n$, subject to the constraints:

$$\left.\begin{aligned} h_j(x)=0, \quad j=1,\ldots,m<n; \\ x_i \ge 0, \quad i=1,\ldots,n. \end{aligned}\right\} \tag{1.19}$$

In the first stage of the solution by the method of Lagrange multipliers, we find all stationary points lying in the positive octant of *n*-dimensional space and isolate the maximum points on the basis of sufficient conditions for an extremum. Then we explore the positive octant boundary, in turn equating to

zero in all sorts of combinations of $1, 2, \ldots, n-m+1$ variables, and each time solving the optimization problem with equality constraints. As a result of the computing process, the complexity of which is obvious, the largest of all the extrema should be selected.

A more general problem, find the maximum

$$f(x),\ x \in E^n \tag{1.20}$$

under constraints $h_j(x)=0$, $j=1,\ldots,m$; $g_i(x)\geq 0$, $i=1,\ldots,p$, can be reduced to just considered by the introduction of additional variables y_i , $i=1,\ldots,p$, such that

$$g_i(x)-y_i=0,\quad y_i\geq 0. \tag{1.21}$$

The extremum can be achieved in a region, where $y_i>0$, or at its borders, where $y_i=0, i=1,\ldots,p$. Lagrange function for the constrained optimization problem (1.20), (1.21) has the form

$$L(x,y,\lambda)=f(x)+\sum_{i=1}^{m}\lambda_i h_i(x)+\sum_{i=1}^{p}\lambda_{m+i}\left(g_i(x)-y_i\right).$$

In the optimum point its partial derivatives by x_j, y_j, λ_j vanish, including

$$\frac{\partial L}{\partial y_j}=\hat{\lambda}_{m+j}=0,\quad j=1,\ldots,p.$$

This condition means that if at the point of extremum $\hat{y}_j>0$, then $\hat{\lambda}_{m+j}=0$, on the other hand, if $\hat{y}_j=0$, that is on the border area $\hat{\lambda}_{m+j}\neq 0$, as the corresponding limit should be considered. Thus, the property

$\hat{y}_j \dfrac{\partial L}{\partial y_j} = 0, \quad j = 1, \ldots, p$ is proved. Obviously, the problem (1.20) and (1.21) are completely identical to (1.24) and can be solved in the same way.

For the problem can be written necessary optimality conditions (generalized Lagrange multiplier rule), however, it is rarely used because of the complexity of solving the resulting system of equations.

1.4.2 Nonlinear Programming

Subject of nonlinear programming

Nonlinear programming – branch of applied mathematics dealing with finding the extremum of function of many variables in the presence of non-linear constraints in the form of equalities and inequalities, i.e. solution of the problem (1.14), discussed in the previous section [3].

Classical methods of optimization are part of it, along with disciplines such as linear, quadratic, separable programming. However, of the greatest practical interest to us are the numerical or direct methods of nonlinear programming, especially intensively developed in recent years.

None of the proposed algorithms is absolutely the best, so the choice of a numerical method is dictated by the content of a specific problem, which must be solved. Computational methods are classified according to some peculiarity of the problem (no restrictions, with equality constraints, inequalities and so on), the nature of methods of solutions (e.g., with or without the use of derivatives), the type of computers, programming language, and so on.

Search of one variable function extremum

A number of methods of finding an extremum of function of many variables use as a part the procedure for the one-dimensional optimization. In the case then function of one variable is multi-extremal, the only correct method of

finding the global extremum is a direct enumeration of a number of values with some step in its change.

Obviously, the function can vary sharply, the smaller should be chosen the grid. After a rough determination of the neighborhood of extremum, begin to search its exact value. For this purpose, one-dimensional algorithms for searching the extremes of unimodal functions in a given interval are used.

One of the most effective methods is the so-called golden section. Recall that if a segment divided into two parts, so that the ratio of the lengths at a greater relative length equal to the length of most of all segment, obtain the so-called golden ratio (is approximately 0.38: 0.62). Golden section method just based on the multiple division of uncertainty interval, i.e. the interval in which the extremum enclosed in an appropriate ratio.

Suppose that in some approximation known interval Δ_i in which the function extremum exists. Divide it by points y_1^i, y_2^i in the proportion of the golden section. If $y_2^i > y_1^i$ we discarded x_2^i, indicating $x_2^{i+1} = y_2^i$ a segment Δ_{i+1} share in the proportion of the golden section, and so on. To reduce the range of uncertainty in the 100 times 11 calculations is required, in the 10000 times – 21 calculations. For comparison, the bisection method (dichotomy) leads to a corresponding narrowing of the range of 14 and 28 function evaluations.

The advantage of the golden section is that it works equally well for smooth and non-smooth functions. It was found that, in the case of smooth functions by a polynomial approximation possible to quickly determine the number of extreme at the same accuracy as that by the golden section.

If the optimized function is defined and unimodal on the entire real axis, there is no need to worry about selecting the initial uncertainty interval. For example, in the method of Davis, Sven and Campy (abbreviated as DSC), from a certain

point, it becomes increasing steps until extremum is passed, and then made quadratic interpolation on the basis of information about the functions in the past three points is determined extremum of the interpolation polynomial.

The *Powell's algorithm* of quadratic polynomial interpolation is carried out in three arbitrary points, approximate extremum is found, dropped one of the four points and the procedure is repeated until convergence. The most effective is a combination of the described algorithms, or the so-called method of *DSC-Powell*. In accordance with this first algorithm DSC sought interval in which the extremum, three points are selected and carried there through parabola. Approximate value at an extremum is calculated as in the method of Powell:

$$\bar{x} = \frac{1}{2}\frac{\left(x_2^2 - x_3^2\right) f\left(x_1\right) + \left(x_3^2 - x_1^2\right) f\left(x_2\right) + \left(x_1^2 - x_2^2\right) f\left(x_3\right)}{\left(x_2 - x_3\right) f\left(x_1\right) + \left(x_3 - x_1\right) f\left(x_2\right) + \left(x_1 - x_2\right) f\left(x_3\right)}.$$

If the value of the function at the point $\bar{x}$ of optimum values $f\left(x_1\right)$, $f\left(x_2\right)$, $f\left(x_3\right)$ differ by less than a predetermined accuracy, complete calculations, otherwise discard the worst of the points $x_1, x_2, x_3, \bar{x}$, and carry out a new parabola. For functions that are sufficiently close to quadratic efficiency DSC-Powell is very high: as a rule, the decision to an accuracy $10^{-5} \ldots 10^{-6}$ is achieved 6–8 calculations of the objective function.

Methods for unconstrained optimization

Consider the problem of finding the maximum of a function of several variables without restrictions. Find maximum $f\left(x\right)$, $x \in E^n$. One of the most famous is the gradient methods to solve this problem. They are based on the fact that the promotion of the objective function to the extreme in the space E^n made by the rule

$$x_{k+1} = x_k + \Delta x_k . \tag{1.22}$$

There Δx_k – transition vector from point x_k to the point x_{k+1}, $\Delta x_k = \lambda_k s_k$, where s_k – the unit vector in the direction Δx_k; λ_k – a scalar.

Vector s_k sets another search direction and λ_k – the length of a step in this direction. Obviously, λ_k should be chosen so as to move as close as possible to the extreme. Various methods of selecting the direction of the search are used. The simplest of these is that the movement of the point x_k is made in the direction of the greatest magnification of $f(x_k)$, i.e. in the direction of a gradient function at a given point.

According to this method, called the method of *steepest descent*,

$$s_k = \frac{\nabla f(x_k)}{\left\| \nabla f(x_k) \right\|},$$

where $\left\| \nabla f(x_k) \right\| = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f(x_k)}{\partial x_i} \right)^2}$, and the formula of the transition from x_k to x_{k+1} has the form

$$x_{k+1} = x_k + \lambda_k \frac{\nabla f(x_k)}{\left\| \nabla f(x_k) \right\|} . \tag{1.23}$$

Consider a geometric interpretation of the steepest descent method in the case of two variables. Transition from formula (1.21) does not allow to come to a point extremum by one step; the procedure should be repeated many times until it reaches a maximum, i.e., conditions $\left\| \nabla f \right\| = 0$ are fulfilled. Partial derivatives of the function calculation at points generally performed numerically. Search step, you can select a constant, but it is better to define it in terms of

$$\max_{\lambda_k} f\left(x_k + \lambda_k s_k\right)$$

using the previously discussed methods of one-dimensional search.

The theory shows, and the practice of calculation confirms that the steepest descent method is not very effective in cases where the level curves of the objective function is strongly stretched, i.e. there are deep ravines while searching a minimum or ranges when searching maximum. The steepest descent direction is almost orthogonal to the best direction of the search, as a consequence, the optimal step reduced all the time, and the algorithm "get stuck" without reaching the extreme. The way out of this situation can be a scaling of variables, at which the level lines would get kind of close to the circle.

In order to reduce the amount of computations of the objective function, associated with a numerical definition of partial derivatives, sometimes used method of *coordinate descent*, which is also called a relaxation or Gauss-Seidel method. Let e_i – axis x_i unit vector, and $x = \{x_1, \ldots, x_n\}$ – the starting point of the search. One iteration of coordinate descent is to take steps: $x_{k+1} = x_k + \lambda_k e_k$, $k = 1, \ldots, n$.

Step as in the method of steepest descent is determined by the condition $\max_{\lambda_k} f\left(x_k + \lambda_k s_k\right)$. The Gauss-Seidel method suffered from the same flaw as the steepest descent method, – a bad convergence in the presence of ravines.

One way to overcome the computational difficulties associated with the gully structure of the objective function involves the use of information not only on its first derivative, but also higher order, contained in the second partial derivatives. An arbitrary function can be represented by its quadratic expansion in a Taylor series in the neighborhood of point *x*:

$$f\left(x+\Delta x\right)=f\left(x\right)+\nabla^{T}f\left(x\right)\Delta x+\frac{1}{2}\left(\Delta x\right)^{T}H\left(x\right)\Delta x.$$

The minimum in the direction Δx is obtained by differentiation for each of the components of the vector Δx, which gives

$$\Delta x=-H^{-1}\left(x\right)\nabla f\left(x\right). \tag{1.24}$$

If we substitute (1.23) into (1.21), we obtain an expression for the minimum point of the quadratic function

$$x=x-H^{-1}\left(x\right)\nabla f\left(x\right). \tag{1.25}$$

In the case where the objective function from the outset is a quadratic, the optimum point is found by one step, but if the function is arbitrary, this fails to achieve the minimum and should be repeatedly use the formula (1.25):

$$x_{k+1}=x_{k}-H^{-1}\left(x_{k}\right)\nabla f\left(x_{k}\right). \tag{1.26}$$

Even better, by analogy with gradients instead of (1.25) to use the relation

$$x_{k+1}=x_{k}+\lambda_{k}s_{k}=x_{k}+\lambda_{k}\frac{H^{-1}\left(x_{k}\right)\nabla f\left(x_{k}\right)}{\left\|H^{-1}\left(x_{k}\right)\nabla f\left(x_{k}\right)\right\|}, \tag{1.27}$$

and the step λ_{k} choose from the $\min\limits_{\lambda_{k}} f\left(x_{k}+\lambda_{k}s_{k}\right)$ minimum condition.

Equations (1.26) or (1.27) are applied iteratively until the end calculation process criterion is reached, called *Newton's method*. Difficulties of using Newton algorithm associated, firstly, with Hessian matrix inversion, and secondly, with the computation of the second partial derivatives, which restricts its practical use.

The methods of conjugate directions are without drawbacks of gradient methods and have the convergence rate close to Newton's method. At the same time, they are the methods of the first order, as the gradient. Positive defined

quadratic form of *n* variables is minimized conjugate gradient method for no more than *n* steps. The conjugate gradient method is suitable for minimization of non-quadratic functions, only when they are iterative.

Two vectors *x*, *y* in the space E^n called conjugate relative to the matrix *H*, if $x^T Hy = 0$. Consider the quadratic function of the *n* variables

$$f(x) = a + x^T b + \frac{1}{2} x^T Hx \tag{1.28}$$

with a positive defined matrix *H*. Let's apply for function $f(x)$ minimization iterative process $x_{k+1} = x_k + \lambda_k s_k$. The direction of descent to *k*-th step is one of the vectors of conjugate vectors $s_0, s_1, \ldots, s_{n-1}$. If you select λ_k from the minimum of $f(x_k + \lambda_k s_k)$, i.e. $\frac{\partial f(x_k + \lambda_k s_k)}{\partial \lambda_k}$, that, differentiating by step (1.27), we obtain

$$\lambda_k = -\frac{\nabla^T f(x) s_k}{s_k^T H s_k}. \tag{1.29}$$

Applying the formula (1.28), (1.29), on n-th step of the iterative process will find

$$x_n = x_0 + \sum_{k=0}^{n-1} \lambda_k s_k = x_0 - \sum_{k=0}^{n-1} \frac{\nabla^T f(x) s_k}{s_k^T H s_k}. \tag{1.30}$$

We can say that the point x_n is the exact minimum of the function $f(x)$, i.e. $x_n = \widehat{x} = -H^{-1} b$, which means that the process (1.30) with the choice of λ_k by (1.28) does give the opportunity to find the minimum of a quadratic function by *n* steps.

There are different ways of constructing conjugate directions. In particular, Fletcher and Reeves proposed a method, called the conjugate gradients method, according to which the subsequent direction of the search is a linear combination of the direction of steepest descent and the previous direction, i.e.,

$$s_k = -\nabla f\left(x_k\right) + \beta_{k-1} s_{k-1}. \tag{1.31}$$

As the initial search direction $s_0 = -\nabla f\left(x_0\right)$ is chosen. The weighting factors β_{k-1} are determined so that the directions $s_0, s_1, \ldots, s_{n-1}$ were conjugated. It can be shown that

$$\beta_{k-1} = \frac{\left\|\nabla f\left(x_k\right)\right\|^2}{\left\|\nabla f\left(x_{k-1}\right)\right\|^2}.$$

Since the direction of the search is conjugated to a quadratic function, the Fletcher-Reeves method leads to the solution of no more than *n* steps. In the case of an arbitrary function is recommended after every *n* steps "upgrade" the search direction by setting $s_n = -\nabla f\left(x_n\right)$ and repeat the process (1.30) with replacement of x_0 to x_n.

Some methods do not use the derivatives of functions, and the optimization direction in which tis determined only on the basis of successive calculations of the objective function. In cases where the determination of the objective function derivatives is difficult, *search algorithms* may be preferable. In the case of one-dimensional analogue of the search method is the method of golden section, and the method of using derivatives – DSC-Powell method.

Methods of optimization with constraints

In addition to the previously described method of Lagrange multipliers for finding the extremum of functions with restrictions a number of numerical

methods developed. The first approach to the construction of algorithms for constrained optimization is monotonous motion to the optimum of the objective function and at the same time striving to meet the exact or approximate limits. Methods of this type are numerous, but the complexity, lack of flexibility and a large amount of computational work limit their use in practical calculations.

More elegant, easy to implement and effective the methods based on the reduction of problems with constraints to the solution of a sequence of unconstrained optimization – the so-called penalty function methods. There are several variations of these methods.

Let's begin their consideration with the interior point method for problems with inequality constraints:

find the maximum $f(x),\ x \in E^n$

with restrictions $g_j(x) \geq 0, \quad j = 1, \ldots, p.$ (1.32)

To determine the conditional extremum built the so-called attached objective function

$$I_k^*(x, \Lambda_k) = f(x) - \Lambda_k \sum_{j=1}^{p} \frac{1}{g_j(x)}, \tag{1.33}$$

where Λ_k – a number, called penalty factor; $\sum_{j=1}^{p} \frac{1}{g_j(x)}$ – penalty function.

The algorithm for solving the problem (1.32) is the following: allowable point x_0 at which everything $g_j(x_0) \geq 0$ is selected, and a monotonically decreasing sequence of positive penalties Λ_k; for every $k = 1, 2, \ldots,$ starting with the point $\hat{x}_{k-1}$, it solves the problem of unconstrained optimization function (1.33).

If for every k it is possible to find the maximum of I_k^* by x, the sequence $\{\hat{x}_k\}$ converges to the solution of the problem (1.33).

The organization of numerical maximum search of (1.29) must be such that the point does not leave the feasible region. This shortage is deprived the *external penalty function method*, which for the problem of the form (1.33) involves the construction of the associated objective function

$$I_k^*(x, \Lambda_k) = f(x) - \Lambda_k \sum_{j=1}^{p} \left(g_j^+(x)\right)^2, \tag{1.34}$$

Where $g_j^+(x) = \min\{0, g_j(x)\}$.

Thus, inside the allowable region, where $g_j(x) \geq 0$, $g_j^+(x) = 0$, and $g_j^+(x) = g_j(x)$ outside.

In contrast to (1.33), the function (1.34) is defined for all $x \in E^n$.

The algorithm for solving the problem is as follows: take an arbitrary point x_0, and monotonically increasing sequence of numbers $\Lambda_k \to \infty$; for $k = 1, 2, \ldots,$ starting from $\hat{x}_{k-1}$, it solves the problem of unconstrained optimization function (1.34), with the result that is determined the new approach $\hat{x}_k$.

It can be shown that the sequence of points $\hat{x}_k$ converges to the solution of the problem (1.33), but in contrast to the interior point method to the extreme movement takes place outside the feasible set, and is taken from the name of the method of exterior penalty functions. This method is also applicable to the general problem of nonlinear programming (1.14), for which used attached objective function

$$I_k^*(x, \Lambda_k) = f(x) - \Lambda_k \left[\sum_{j=1}^{m} h_j^2(x) + \sum_{j=1}^{p} \left(g_j^+(x) \right)^2 \right]. \quad (1.35)$$

Algorithm of solution is the same as for the problem (1.34).

The solution of nonlinear programming problems with constraints using penalty function method is complicated by the fact that as the penalty function coefficient is increasing, (1.35) expressed gully structure. As previously shown, not all the methods of unconditional optimization solution can cope with such problems, and therefore the choice of the method of finding the extremum of the attached objective function is of fundamental importance.

An important role is also played the strategy of the penalty factor change, because if you choose it immediately large, constraints of the problem satisfied well, but the objective function does not improve. In contrast, if too small values of Λ_k, motion occurs in the direction of improvement of the objective function, but practically does not take into account the constraints that can lead to failure in the E^n areas where the objective function and constraints are not defined.

For example, if in the objective function or in limitations members of the form x^a are present, it is unacceptable entering the zone $x \le 0$. Get rid of the zone uncertainty, resulting in the computer calculations for emergencies can sometimes be the introduction of a suitable change of variables. In particular, to meet conditions $x > 0$ the replacement $x = e^z$ is suitable, which already $z \in E^1$. If such a reception is impossible, it should be carefully selected constants of unconditional search methods as the length of the step in the direction of descent, change of this step in the process to find a one-dimensional vector of variables did not leave the area where the objective function and constraints of the problem identified.

In conclusion, we consider the possibility of nonlinear optimization methods usage in order to solve systems of nonlinear equations. Suppose that in the problem (1.14) there are no restrictions in the form of inequalities, and the number of variables equal to the number of restrictions in the form of equations, i.e., in fact, the task of solving the system of *m* equations with *m* unknowns. We form the function

$$I^* = -\sum_{j=1}^{m} h_j^2(x) \tag{1.36}$$

and find its maximum. If the system of equations $h_j(x) = 0$, $j = 1, \ldots, m$, has a solution, then, obviously, at the same time with the maximum of I^* is the root of the system of equations. In particular, if the functions $h_j(x)$ are linear, function (1.36) is obtained quadratic and can be effectively solved by Newton's and conjugate gradient method.

Replacement of the problem of systems of linear equations solution to extremum problems is justified in cases where the matrix of the system is ill-conditioned (e.g., in the problem of approximation by least squares) and can not be solved by conventional methods, in particular, by process of elimination.

The values $h_j(x)$ in (1.35) are called residuals, and the solution of nonlinear equations is replaced by *minimizing the sum of squared residuals*.

1.4.3 Methods for Optimization of Hardly Computable Functions

In some problems, when the calculation of the value of the objective function may take minutes, hours or even days of the computer, the range of acceptable methods of optimization significantly narrowed.

These problems, in particular, include aerodynamic optimization of turbine blades using CFD.

The Nelder-Mead method (Nelder A.-Mead R.), also known as the flexible polyhedron method or the simplex method is a method of unconditional optimization of functions of several variables. Without requiring computation of the gradient function, it is applicable to non-smooth, noisy functions, and is particularly effective in small (up to 6) number of variable parameters. Its essence lies in the follow-successive movement and deformation of the simplex around the point of extreme. The method is a local extremum and can "get stuck" in one of them. If you still need to find a global extremum, one can try to select other initial simplex.

A more developed approach to the exclusion of local extrema offered algorithm based on the Monte-Carlo method, as well as evolutionary algorithms.

The genetic algorithm (GA) – is a global search heuristic method, used to solve optimization problems and modeling, by random selection, combination and variation of the required parameters with the use of mechanisms that resemble biological evolution. GA usage assumes its careful adjustment on special test functions, which, however, does not guarantee the effectiveness of the algorithm and the accuracy of decisions of the function.

This algorithm is well suited to the study of noisy functions, but requires a large number of CFD – calculations and therefore more time on optimization. The last forcing researchers to use coarse meshes and not quite accurate, but easily calculated turbulence models, which will inevitably leads to loss of the numerical calculations precision.

Monte-Carlo (random search) methods allows you to find the extremes of multimodal and noisy functions; use various constraints during optimization; is particularly effective when a large number of variable parameters; requires careful adjustment for test functions; it is one of the most common methods of optimization and solution of various problems in mathematics, physics, economics, etc. However, the method requires tens of thousands of the objective

function computing and practically not applicable for direct optimization based on CFD – calculations. To improve the efficiency of random search used quasi-random sequence of numbers (LPτ [4] Sobol), Faure, Halton et al.). Increased efficiency is achieved by eliminating clustering that occurs in a random search that is by more even distribution of points in the search study area of the function extremum.

Recently, in the optimization algorithms the methods of experiment planning are widely used. Using the methods of the theory of experimental design (Design of the Experiment – DOE), the original mathematical model can be approximated by a quadratic polynomial. One of the relevant planning schemes of the experiment described in Section 1.3. These quadratic polynomials can be used to further optimization with the use of a universal and reliable global search method using a quasi-random sequences.

1.5 The Practice of Numerical Methods Usage for Local Leveled Optimization Problems Solution

To solve demanded by practice of axial turbines design multi-criteria problems, multi-parameter and multi-mode optimization of the multistage flow path further development and improvement of appropriate numerical methods and approaches required.

It should be noted some features of numerical solution of problems related to the optimization of design objects based on their modes of operation, multimodal objective functions, as well as issues related to the multi-objective optimization problems.

Some aspects of the above problems solutions are given below.

1.5.1 Solution of the Multi-Criteria Optimization Problems

Set out in section 1.4 are the basic optimization techniques. However, depending on the formulation of the optimization problem, as well as the selected design object there are some features of numerical implementation of these methods and their applications.

It is known that the actual design object is usually characterized by a number of quality indicators and improvement in one of them leads to a deterioration in values of other quality criteria (Pareto principle). In such cases it is necessary to consider the optimization problem from many criteria.

The authors offer a well-established practice in solving multi-objective optimization problems – "convolution" of partial objective function weighted by μ_i depending on the importance of a particular quality criteria in a comprehensive quality criteria based on the following

$$\left\| Y^*\left(\vec{x}_d, \vec{x}_p\right)\right\| = \sqrt{\sum_{i=1}^{n}\left(\mu_i Y_i^*\left(\vec{x}_d, \vec{x}_p\right)\right)^2}, \tag{1.37}$$

where Y_i^* – the components of the vector criterion (partial indicators of quality of the object); $\vec{x}_d, \vec{x}_p$ vectors of design parameters and operational parameters, respectively, which together define a design decision.

In fact, (1.37) is the magnitude of the partial criteria of quality, taking into account their weights (μ_i).

Thus, in the *n*-dimensional normalized criterial space each variant of definitely best design object is characterized by a corresponding so-called Pareto point, whose distance to the center of coordinate proportional to the value of the module $\left\| Y^*\left(\vec{x}_d, \vec{x}_p\right)\right\|$ of vector quality criterion.

The experience of steam turbines cylinder optimization with the flow extraction for the purposes of regeneration and heating shows that there is needed to consider at least two criteria of quality – the efficiency of the flow of the cylinder and the power, generated by them.

1.5.2 The Numerical Solution of the Optimization Problem with the Multimodal Objective Function

In some cases it is necessary to check the objective function on multimodality.

In the developed subsystem of multi-criterial and multi-level multi-parameter optimization of design objects to find the optimal solution the search is always performed in two stages whether unimodal or multimodal objective function.

Thus, the first (preliminary) stage is used to determine suspicious extremum points, to find which method is used ideas swarm (Bees Algorithm), the first work of which were published in 2005 [5, 6]. The method is an iterative heuristic multi-agent random search procedure, which simulates the behavior of bees when looking for nectar.

The criterion for the selection of points and their respective sub-areas, in which will be specified by the relevant decision of optimization problems, is the Euclidean distance $R_{ab} = \left\| \vec{x}_a - \vec{x}_b \right\|$ in the space of optimized parameters between the compared points from the set LPτ sequence.

If the Euclidean distance R_{ab} between two points of LPτ sequence $\left(\vec{x}_a, \vec{x}_b \right)$, less than some fixed value R_{set}, then point with the large value of the objective function is selected.

Criteria evaluation for quality and functional limitations at the preliminary stage is performed by using FMM (of the form (1.2) or (1.12)). After processing

all of the set of LPτ sequence points by a "swarm" algorithm suspicious extremum point are defined.

These points are then used as initial approximations of the final (refining) stage of the optimal solution finding. When refining the optimal solutions around the extremum suspicious spot, in a recursive optimize algorithm it is provided the transition from the evaluation criteria of quality and functional limitations by using FMM to their evaluation by appropriate OMM. It uses a method of coordinate descent or conjugate gradient method, for example, Fletcher-Reeves. Thus found several points of local optima are sorted by the value of the objective function, and the best solution given the status of optimal.

1.5.3 The Method of Optimization Taking into Account Turbine Operating Modes

The above (1.37) convolution vector type of the objective function allows to take into account the specific feature of the problem of optimal design of facilities intended for use as a constant, and the variable modes. In the case of optimization taking into account the variability of operating loads, function (1.37), on the one hand, carries information about the overall effectiveness of the design in all modes of operation, and on the other hand, it emphasizes the Pareto signs of the competitive effect of 'individual' quality criteria for each of the operating modes on the final result.

Below is a description of the developed method, which provides the solution of problems of optimum design of turbomachinery, operated at a predetermined range of modes.

This method is based on the integration of formal macromodels of the objective functions.

When included in the examination of the alleged operation modes, created FMM criteria of quality and functional limitations are functions of the design

and operational parameters. Ranges of change of regime parameters are selected in accordance with the proposed schedule changes and they do not change in the course of iterations to refine the optimal solutions.

Such FMM usage at the step of finding the optimal solutions necessitates multiple evaluation of quality criteria and functionality limitations for each sampling point (corresponding to a combination of structural parameters), the number of calculations of each FMM considered equivalent to the number of operating modes. Obviously, the increased number of calculations requires additional computing resources in the search for the best design.

The decision of the problem marked can be achieved by eliminating the regime parameters of the vector of varied FMM parameters (1.2). To eliminate the regime parameters it is necessary to carry out the FMM integration. In this case, the new FMM coefficients of integral quality criterion obtained from the following relationship:

$$Y(q) = A_0 + \sum_{i=1}^{N_c} A_i q_i + \sum_{j=1}^{N_m} A_j \int_0^1 q_j(t)\,dt + \sum_{i=1}^{N_c-1} \sum_{k=i+1}^{N_c} A_{ik} q_i q_k + \\ + \sum_{j=1}^{N_m-1} \sum_{m=j+1}^{N_m} A_{jm} \int_0^1 q_j(t) q_m(t)\,dt + \sum_{i=1}^{N_c} \sum_{j=1}^{N_m} A_{ij} q_i \int_0^1 q_j(t)\,dt + \\ + \sum_{i=1}^{N_c} A_{ii} q_i^2 + \sum_{j=1}^{N_m} A_{jj} \int_0^1 q_j^2(t)\,dt, \qquad (1.38)$$

where N_c, N_m – numbers of structural and operational parameters, respectively; t – time.

The new FMM of form (1.38) contains integrals of regime parameters, which can be calculated from the charts of regime parameters $\left(q_j(t)\right)$ and converted to the form:

$$Y_m(q) = A_{0m} + \sum_{i=1}^{N_c} \left(A_{im} q_i + A_{ii} q_i^2 \right) + \sum_{i=1}^{N_c-1} \sum_{k=i+1}^{N_c} A_{ik} q_i q_k, \qquad (1.39)$$

where

$$\left.\begin{aligned} A_{0m} &= A_0 + \sum_{j=1}^{N_m}\left(A_j \int_0^1 q_j(t)\,dt + A_{jj}\int_0^1 q_j^2(t)\,dt \right) + \\ &+ \sum_{j=1}^{N_m-1}\sum_{m=j+1}^{N_m} A_{jm}\int_0^1 q_j(t)\,q_m(t)\,dt; \\ A_{im} &= A_i + \sum_{j=1}^{N_m} A_{ij}\int_0^1 q_j(t)\,dt. \end{aligned}\right\} \tag{1.40}$$

FMM form (1.39) is more convenient to use in the optimization algorithms for quality criteria and functional constraints evaluation, as presented macromodel depends only on the design parameters that do not change their values when changing the operating mode of the FP. Thus, the account of the expected schedule change duty operation is performed due to the fact, that the operating parameters are integrally included in the new coefficients FMM (1.40).